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Araştırma Makalesi / Research Article

Construction of Soliton Solutions for Chaffee-Infante Equation

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Keywords
Chaffee–Infante equation; Sine-Gordon expansion method; Soliton solutions; Exact solutions

Abstract

In this article, has been studied on the Chaffee-Infante equation and soliton solutions of these equation are examined. In accordance with this purpose, The sine-Gordon expansion method, which is one of the solution methods of nonlinear partial differential equations, was used. Also graphical representation of the obtained results of the specified equation is made using Wolfram Mathematica 12 for certain values and thus the conformity of the founded results has been demonstrated.

Chaffee-Infante Denklemi için Soliton Çözümlerinin Oluşturulması

Anahtar kelimeler
Chaffee–Infante denklemi; Sine-Gordon açılım metodu; Soliton çözümler; Tam çözümler

Öz

Bu makalede, Chaffee-Infante denklemi üzerinde çalışılmıştır ve bu denklemin soliton çözümleri incelenmiştir. Bu amaç doğrultusunda, lineer olmayan kısmi diferansiyel denklemlerin çözüm yöntemlerinden biri olan sine-Gordon açılım yöntemi kullanılmıştır. Ayrıca belirtilen denklemin elde edilen sonuçlarının grafiksel gösterimi belli değerler için Wolfram Mathematica 12 programı kullanılarak yapılmış ve böylece bulunan sonuçların uygunluğu gösterilmiştir.

1.Introduction

The nonlinear evolution equations (NLEEs) have so important usage areas in many areas such as physics, chemistry, biology, optics, fluid dynamics, hydro magnetic waves and many others. Recently, various methods have been developed by many researchers for NLEEs, which have many uses in mathematics and physics (Akram and Mahak 2018, Alam and Akbar 2014, Qawasmeh and Alquran 2014, Taghizadeh 2012, Tasbozan *et al*. 2016, Tuluce Demiray *et al*. 2015, Tuluce Demiray and Bulut 2017a, Tuluce Demiray and Bulut 2017b, Tuluce Demiray and Bulut 2019, Wazwaz 2005, Durur *et al*. 2020, Durur and Yokuş 2021, Yokuş *et al*. 2021, Duran 2021). In this study, we will use the sine-Gordon expansion method (SGEM), which is one of the widely used methods to find the solutions of NLEEs (Bulut *et al*. 2016, İlhan *et al*. 2020, Kumar *et al*. 2017). SGEM has been created from traveling wave transformation and sine-Gordon equation (Yan and Zhang 1999). We will use SGEM to find the soliton solutions of the Chaffe-Infante equation (CIE). CIE is given as:

$$u_{xt}+\left(-u_{xx}+au^{3}+au\right)_{x}+mu_{yy}=0. \tag{1}$$

Where a and m arbitrary constants. CIE is a famous reaction duffing equation and used in environmental science, fluid dynamics electronic high-energy physic and so on. Solutions of CIE given by Eq. (1) have been tried recently to be found by many researchers with various methods such as Habiba *et al*. used the improved Kudryashov method (Habiba *et al*. 2019). Tahir *et al*. used the generalized Kudryashov method (Tahir *et al*. 2020). Sakthivel and Chun used the exp-function method (Sakthivel and Chun *et al*. 2010). Akbar *et al*. used the first integral method (Akbar *et al*. 2019). Mao used the trial equation method and canonical-like transformation method (Mao *et al*. 2018). Our aim in this article is to find the solutions of the CIE through SGEM. In Section 2, SGEM's basic structure is given. In Section 3, Applying SGEM to the CIE some soliton solutions of the equation is founded.

2. Basic structure of SGEM

In this chapter we will give the common structure of SGEM. First, we first take into account the sine-Gordon equation

$$v_{xx} - v_{tt} = m^2 \sin(v). \tag{2}$$

Where $v = v(x,t)$ and m is a real constant.

Performing the wave transformation $v(x,t) = V(\xi), \xi = \mu(x - kt)$ to Eq. (2), a nonlinear ordinary differential equation is obtained as follows.

$$V'' = \frac{m^2}{\mu^2\left(1-k^2\right)} \sin(V). \tag{3}$$

Integrating Eq. (3) and equalling the integration constant to zero. We get the following equation.

$$\left[\left(\frac{V}{2}\right)'\right]^2 = \frac{m^2}{\mu^2\left(1-k^2\right)} \sin^2\left(\frac{V}{2}\right). \tag{4}$$

Substituting $\varphi(\xi) = \dfrac{V}{2}$ and $b^2 = \dfrac{m^2}{\mu^2\left(1-k^2\right)}$ in Eq. (4), we get:

$$\varphi' = b\sin(\varphi). \tag{5}$$

If we take $b = 1$, we get:

$$\varphi' = \sin(\varphi). \tag{6}$$

From the Eq. (6), we get the following equations.

$$\sin(\varphi) = \sin\left(\varphi(\xi)\right) = \left.\frac{2de^{\xi}}{p^2 e^{2\xi} + 1}\right|_{d=1} = \sec h(\xi), \tag{7}$$

$$\cos(\varphi) = \cos\left(\varphi(\xi)\right) = \left.\frac{d^2 e^{2\xi} - 1}{p^2 e^{2\xi} + 1}\right|_{d=1} = \tanh(\xi), \tag{8}$$

where d is considered as the integral steady.

In order to the find solution of the following nonlinear partial differential equation;

$$F(u, u_x, u_t, u_{xx}, u_{tt}, u_{xt}, u_{xxx}, \ldots), \tag{9}$$

we handle the equation given below,

$$V(\xi) = \sum_{i=1}^{n} \tanh^{i-1}(\xi)\left[B_i \sec h(\xi) + A_i \tanh(\xi)\right] + A_0. \tag{10}$$

Considering the Eqs. (7) and (8), we can write the Eq. (10) as follows:

$$V(\varphi) = \sum_{i=1}^{n} \cos^{i-1}(\varphi)\left[B_i \sin(\varphi) + A_i \cos(\varphi)\right] + A_0. \tag{11}$$

Here we determine the value of n in Eq. (11) by means of the balance principle, replace Eq. (11) into Eq. (9) and comparison the terms. Thus we get a system of equations. Solving this obtained system of equations, we obtain results in moving wave solutions of the Eq. (9).

3 Application of SGEM to the CIE

We the following transformation to the CIE given in Eq. (1)

$$u(x, y, t) = u(\xi), \xi = kx + ly + wt. \tag{12}$$

Applying the transformation Eq. (12), we obtain the ordinary differential equation.

$$kwu''-k^3u'''+3aku^2u'-aku'+ml^2u''=0. \quad (13)$$

In Eq. (13) we integrate with respect of ξ and by taking the integration constant as zero, we obtain.

$$-k^3u''+\left(kw+ml^2\right)u'+ak\left(u^3-u\right)=0. \quad (14)$$

Balancing the terms u'' and u^3. We find $N=1$. Using the value of $N=1$ in Eq. (11), we get:

$$u(\varphi)=B_1\sin(\varphi)+A_1\cos(\varphi)+A_0, \quad (15)$$

$$u'(\varphi)=B_1\cos(\varphi)\sin(\varphi)-A_1\sin^2(\varphi), \quad (16)$$

$$u''(\varphi)=B_1\cos^2(\varphi)\sin(\varphi)- B_1\sin^3(\varphi)-2A_1\sin^2(\varphi)\cos(\varphi). \quad (17)$$

Placing Eq. (15), (16) and (17) into Eq. (14), we generating trigonometric equations. We obtain an equation system by performing some mathematical operations in these trigonometric equations. Solving the obtained system of equations with the help of Wolfram Mathematica Release 12, we can result:

Case1:

$$A_0=-\frac{1}{2}, A_1=-\frac{1}{2}, a=0, k=0, l=0. \quad (18)$$

We get:

$$u_1(x,y,t)=-\frac{1}{2}+\mathrm{sech}[kx+ly+wt]B_1 -\frac{\tanh[kx+ly+wt]}{2}. \quad (19)$$

Case2:

$$A_0=0, A_1=1, B_1=0, a=2k^2, w=-\frac{l^2m}{k}. \quad (20)$$

We get:

$$u_2(x,y,t)=\tanh[kx+ly+wt]. \quad (21)$$

Case3:

$$A_0=-\frac{1}{2}, A_1=-\frac{1}{2}, a=0, k=0, m=0. \quad (22)$$

We get:

$$u_3(x,y,t)=\frac{1}{2}\left(-1-\tanh[kx+ly+wt]\right). \quad (23)$$

Case4:

$$A_0=\frac{1}{2}, A_1=-\frac{1}{2}, B_1=0, \quad a=8k^2, l=-\frac{\sqrt{6k^3-kw}}{\sqrt{m}}. \quad (24)$$

We get:

$$u_4(x,y,t)=\frac{1}{2}\frac{\left(1-\tanh\left[\frac{kx+ly+wt}{2}\right]\right)^2}{1+\tanh^2\left[\frac{kx+ly+wt}{2}\right]}. \quad (25)$$

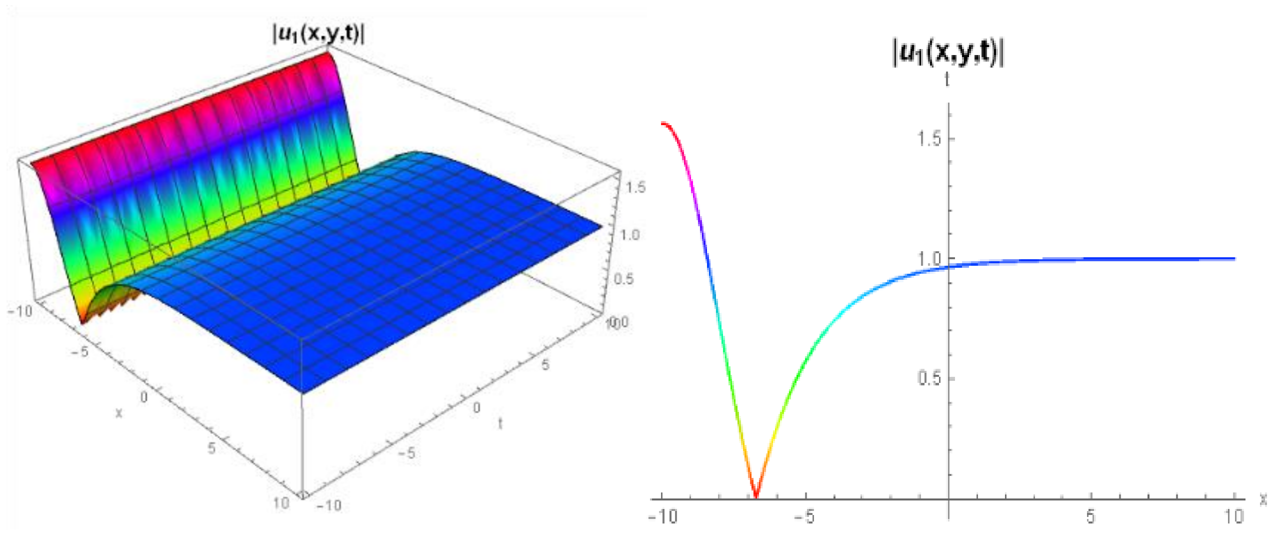

Figure 1. The 3D graph of the Eq. (19) for $k = 0.5, l = 0.25, w = 3, B_1 = 2, y = 1,\ -10 < x < 10, -10 < t < 10$ and 2D graph for this values and $t = 1.5$

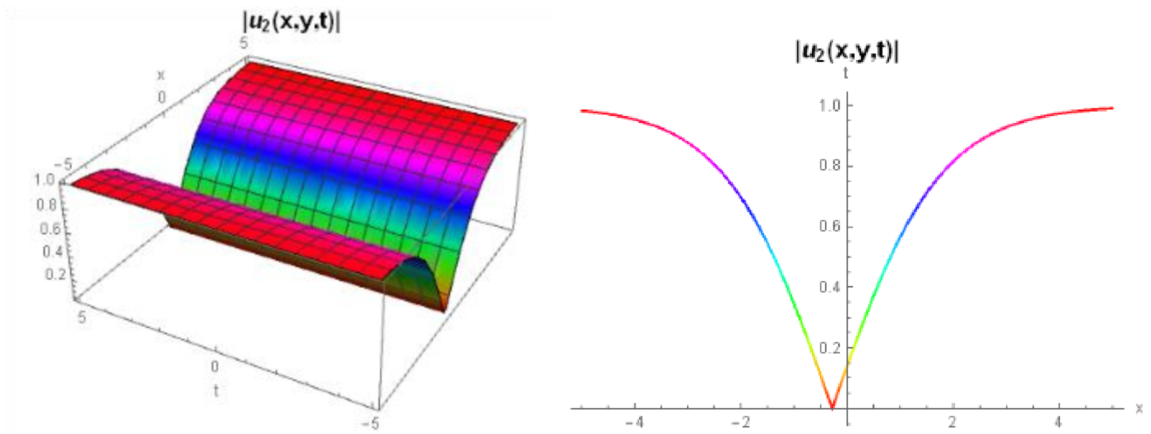

Figure 2. The 3D graph of the Eq. (21) for $k = 0.5, l = 0.8, w = 1, y = 0.05,\ -5 < x < 5, -5 < t < 5$ and 2D graph for this values and $t = 0.1$.

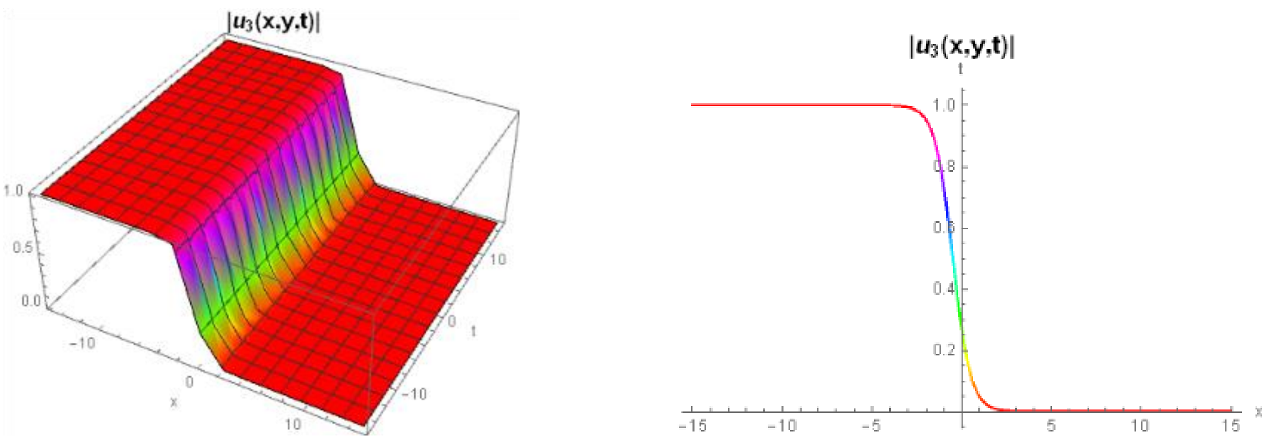

Figure 3. The 3D graph of the Eq. (23) for $k = 1, l = 0.2, w = 0.5, y = 2,\ -15 < x < 15, -15 < t < 15$ and 2D graph for this values and $t = 0.2$.

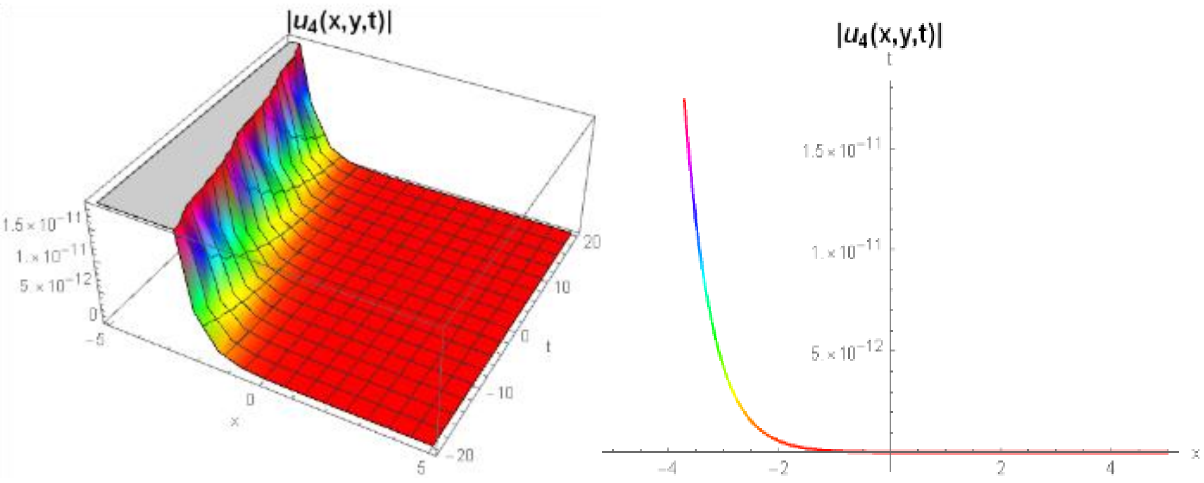

Figure 4. The 3D graph of the Eq. (25) for $k=1, l=8, w=0.05, y=2, \ -5<x<5, -20<t<20$ and 2D graph for this values and $t=2$.

4. Discussions

By applying SGEM to the CIE equation, we found some soliton solutions of the equation. Thus, it has been seen that SGEM is a method that provides effective and precise results. In addition, this method is a suitable tool for solving difficult and complex problems encountered in the solution of NLEEs more easily. When we compare the solutions we obtained in this study with previous studies, our (21) solution is similar to the (26) solution given by Sakthivel and Chun and the (23) and (58) solutions given by Akbar *et al*. In addition our (23) solution is similar to the (25) solution given by Sakthivel and Chun, the (33) and (48) solutions given by Akbar *et al*. and the (14), (16) and (18) solutions given by Tahir *et al*. According to our research our (19) and (25) solutions are not given before and are new.

5. Conclusions

In this work, we obtained the soliton solutions of the CIE by applying SGEM. Thus, we obtained new soliton solutions of the CIE. We drew the 2D and 3D graphical representations of these solitons with the help of a Wolfram Mathematica 12. In this way, we checked the correctness of the solutions we found. As far as we know, SGEM has not been applied to the CIE before. The solutions we obtained have not been presented in previous studies and are new. In the light of the results we have achieved, we consider the sine-Gordon expansion method as an effective method in calculation of NLEEs.

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