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Multipliers for bounded $\mathcal{I}_2$-convergence of double sequences

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ABSTRACT

Multipliers and factorizations for bounded statistically convergent sequences were studied by Connor et al. [J. Connor, K. Demirci, C. Orhan, Multipliers and factorizations for bounded statistically convergent sequences, Analysis 22 (2002) 321–333] and for bounded $\mathcal{I}$-convergent sequences by Yardımcı [Ş. Yardımcı, Multipliers and factorizations for bounded $\mathcal{I}$-convergent sequences, Math. Commun., 11 (2006) 181–185]. In this paper, we get analogous results of multipliers for bounded $\mathcal{I}_2$-convergent double sequences.

1. Introduction

The concept of convergence of a sequence of real numbers has been extended to statistical convergence independently by Fast [1] and Schoenberg [2]. A lot of developments have been made in this area after the works of Šalát [3] and Fridy [4,5]. In general, statistically convergent sequences satisfy many of the properties of ordinary convergent sequences in metric spaces [1,4–6]. This concept was extended to the double sequences by Mursaleen and Edely [7]. Çakan and Altay [8] presented multidimensional analogues of the results of Fridy and Orhan [9].

The idea of $\mathcal{I}$-convergence was introduced by Kostyrko et al. [10] as a generalization of statistical convergence which is based on the structure of the ideal $\mathcal{I}$ of subset of the set of natural numbers. Nuray and Ruckle [11] independently introduced the same idea with another name generalized statistical convergence. Kostyrko et al. [12] gave some of the basic properties of $\mathcal{I}$-convergence and dealt with extremal $\mathcal{I}$-limit points. Das et al. [13] introduced the concept of $\mathcal{I}$ and $\mathcal{I}^*$-convergence of double sequences in a metric space and studied some properties of this convergence. Also, Das and Malik [14] introduced the concepts of $\mathcal{I}$-limit points, $\mathcal{I}$-cluster points and $\mathcal{I}$-limit superior and $\mathcal{I}$-limit inferior of double sequences. Nabiev et al. [15] proved a decomposition theorem for $\mathcal{I}$-convergent sequences and introduced the notions of $\mathcal{I}$-Cauchy sequence and $\mathcal{I}^*$-Cauchy sequence, and then studied their certain properties. A lot of developments have been made in this area after the works of [16–19].

Connor et al. [20] studied multipliers and factorizations for bounded statistically convergent sequences. Also Yardımcı [21] studied multipliers for bounded $\mathcal{I}$-convergent sequences. In this paper, we study multipliers for bounded $\mathcal{I}_2$-convergence of double sequences.

2. Definitions and notations

Throughout the paper, $\mathbb{N}$ denotes the set of all positive integers, χ_A-the characteristic function of $A \subset \mathbb{N}$ and $\mathbb{R}$ the set of all real numbers. We often regard χ_A as sequence (x_{mn}), where $x_{mn} = \chi_A(m, n), A \subset \mathbb{N} \times \mathbb{N}$; note in particular, that e can be regarded as the sequence of all 1's.

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Now, we recall the concepts of convergence, statistical and ideal convergence of the sequences (see [13,1,10,7,22,18, 23–27]).

A double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ of real numbers is said to be bounded if there exists a positive real number M such that $|x_{mn}| < M$, for all $m, n \in \mathbb{N}$. That is

$$\|x\|_\infty = \sup_{m,n} |x_{mn}| < \infty.$$

A double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ of real numbers is said to be convergent to $L \in \mathbb{R}$ if for any $\varepsilon > 0$, there exists $N_\varepsilon \in \mathbb{N}$ such that $|x_{mn} - L| < \varepsilon$, whenever $m, n > N_\varepsilon$. In this case, we write

$$\lim_{m,n\to\infty} x_{mn} = L.$$

By ℓ^2_∞, $c^2(b)$ and $c^2_0(b)$, we denote the space of all bounded, bounded convergent and bounded null double sequences, respectively.

Let $K \subset \mathbb{N} \times \mathbb{N}$. Let K_{mn} be the number of $(j, k) \in K$ such that $j \leq m$, $k \leq n$. If the sequence $\left\{\frac{K_{mn}}{m.n}\right\}$ has a limit in Pringsheim's sense, then we say that K has double natural density and is denoted by

$$d_2(K_{mn}) = \lim_{m,n\to\infty} \frac{K_{mn}}{m \cdot n}.$$

A double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ of real numbers is said to be statistically convergent to $L \in \mathbb{R}$ if for any $\varepsilon > 0$ we have $d_2(A(\varepsilon)) = 0$, where

$$A(\varepsilon) = \{(m, n) \in \mathbb{N} \times \mathbb{N} : |x_{mn} - L| \geq \varepsilon\}.$$

Let $X \neq \emptyset$. A class $\mathcal{I}$ of subsets of X is said to be an ideal in X provided:
(i) $\emptyset \in \mathcal{I}$, (ii) $A, B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$, (iii) $A \in \mathcal{I}$, $B \subset A$ implies $B \in \mathcal{I}$.
$\mathcal{I}$ is called a nontrivial ideal if $X \notin \mathcal{I}$.
Let $X \neq \emptyset$. A non empty class $\mathcal{F}$ of subsets of X is said to be a filter in X provided:
(i) $\emptyset \notin \mathcal{F}$, (ii) $A, B \in \mathcal{F}$ implies $A \cap B \in \mathcal{F}$, (iii) $A \in \mathcal{F}$, $A \subset B$ implies $B \in \mathcal{F}$.

Lemma 2.1 ([10]). *If $\mathcal{I}$ is a nontrivial ideal in X, $X \neq \emptyset$, then the class*

$$\mathcal{F}(\mathcal{I}) = \{M \subset X : (\exists A \in \mathcal{I})(M = X \setminus A)\}$$

is a filter on X, called the filter associated with $\mathcal{I}$.

A nontrivial ideal $\mathcal{I}$ in X is called admissible if $\{x\} \in \mathcal{I}$ for each $x \in X$.

A nontrivial ideal $\mathcal{I}_2$ of $\mathbb{N} \times \mathbb{N}$ is called strongly admissible if $\{i\} \times \mathbb{N}$ and $\mathbb{N} \times \{i\}$ belong to $\mathcal{I}_2$ for each $i \in N$. Throughout the paper, we take $\mathcal{I}_2$ as a strongly admissible ideal in $\mathbb{N} \times \mathbb{N}$.

Let $\mathcal{I}^0_2 = \{A \subset \mathbb{N} \times \mathbb{N} : (\exists m(A) \in \mathbb{N}), \big(i, j \geq m(A) \Rightarrow (i, j) \notin A\big)\}$. Then $\mathcal{I}^0_2$ is a nontrivial strongly admissible ideal and clearly an ideal $\mathcal{I}_2$ is strongly admissible if and only if $\mathcal{I}^0_2 \subset \mathcal{I}_2$.

Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. A double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ of real numbers is said to be $\mathcal{I}_2$-convergent to $L \in \mathbb{R}$, if for any $\varepsilon > 0$ we have

$$A(\varepsilon) = \{(m, n) \in \mathbb{N} \times \mathbb{N} : |x_{mn} - L| \geq \varepsilon\} \in \mathcal{I}_2.$$

In this case, we say that x is $\mathcal{I}_2$-convergent and we write

$$\mathcal{I}_2 - \lim_{m,n\to\infty} x_{mn} = L.$$

If $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ is a strongly admissible ideal, then usual convergence implies $\mathcal{I}_2$-convergence.

A double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ of real numbers is said to be $\mathcal{I}^*_2$-convergent to $L \in \mathbb{R}$ if there exists a set $M \in \mathcal{F}(\mathcal{I}_2)$ (i.e., $\mathbb{N} \times \mathbb{N} \setminus M \in \mathcal{I}_2$) such that

$$\lim_{m,n\to\infty} x_{mn} = L$$

for $(m, n) \in M$ and we write

$$\mathcal{I}^*_2 - \lim_{m,n\to\infty} x_{mn} = L.$$

A double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ of real numbers is said to be $\mathcal{I}_2$-bounded if there exists a real number $M > 0$ such that

$$\{(m, n) \in \mathbb{N} \times \mathbb{N} : |x_{mn}| \geq M\} \in \mathcal{I}_2.$$

We say that an admissible ideal $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ satisfies the property (AP2) if for every countable family of mutually disjoint sets $\{A_1, A_2, \ldots\}$ belonging to $\mathcal{I}_2$, there exists a countable family of sets $\{B_1, B_2, \ldots\}$ such that $A_j \cap B_j \in \mathcal{I}^0_2$, i.e., $A_j \cap B_j$ is included in the finite union of rows and columns in $\mathbb{N} \times \mathbb{N}$ for each $j \in \mathbb{N}$ and $B = \bigcup_{j=1}^\infty B_j \in \mathcal{I}_2$ (hence $B_j \in \mathcal{I}_2$ for each $j \in \mathbb{N}$).

Now we begin with quoting the lemmas due to Das et al. [13] and Kumar [17] which are needed throughout the paper.

Lemma 2.2 ([13, Theorem 1]). *Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. If $\mathcal{I}_2^* - \lim_{m,n\to\infty} x_{mn} = L$, then $\mathcal{I}_2 - \lim_{m,n\to\infty} x_{mn} = L$.*

Lemma 2.3 ([13, Theorem 3]). *If $\mathcal{I}_2$ is an admissible ideal of $\mathbb{N}\times\mathbb{N}$ having the property (AP2) and (X,ρ) is an arbitrary metric space, then for an arbitrary double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ of elements of X, $\mathcal{I}_2 - \lim_{m,n\to\infty} x_{mn} = L$ implies $\mathcal{I}_2^* - \lim_{m,n\to\infty} x_{mn} = L$.*

Lemma 2.4 ([17, Proposition 3.3]).
(a) *Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. If $\lim_{m,n\to\infty} x_{mn} = L$, then $\mathcal{I}_2 - \lim_{m,n\to\infty} x_{mn} = L$.*
(b) *If $\mathcal{I}_2 - \lim_{m,n\to\infty} x_{mn} = L$ and $\mathcal{I}_2 - \lim_{m,n\to\infty} y_{mn} = K$, then*

(i) $\mathcal{I}_2 - \lim_{m,n\to\infty}(x_{mn} + y_{mn}) = L + K$;

(ii) $\mathcal{I}_2 - \lim_{m,n\to\infty}(x_{mn}y_{mn}) = LK$.

3. Multipliers

In this section, we deal with the multipliers on or into $F_{\mathcal{I}_2}(b)$ and $F^0_{\mathcal{I}_2}(b)$. By $F_{\mathcal{I}_2}$ and $F_{\mathcal{I}_2}(b)$, we denote the set of all $\mathcal{I}_2$-convergent double sequences and both bounded and $\mathcal{I}_2$-convergent double sequences, respectively. And by $F^0_{\mathcal{I}_2}(b)$, we denote the set of all both bounded and null $\mathcal{I}_2$-convergent double sequences.

Definition 3.1. Let E and F be two double sequence spaces. A multiplier from E into F is a sequence $u = (u_{mn})_{m,n\in\mathbb{N}}$ such that

$$ux = (u_{mn}x_{mn}) \in F$$

whenever $x = (x_{mn})_{m,n\in\mathbb{N}} \in E$. The linear space of all such multipliers will be denoted by $m(E,F)$. Bounded multipliers will be denoted by $M(E,F)$. Hence we write

$$M(E,F) = \ell^2_\infty \cap m(E,F).$$

If $E = F$, then we write $m(E)$ and $M(E)$ instead of $m(E,F)$ and $M(E,F)$, respectively.

Theorem 3.2. *If E and F are subspaces of ℓ^2_∞ that contain $c^2_0(b)$, then*

$$c^2_0(b) \subset m(E,F) \subset \ell^2_\infty.$$

Proof. The first inclusion follows from noting that if $u \in c^2_0(b)$ and $x \in E \subset \ell^2_\infty$, then we have

$$ux \in c^2_0(b) \subset F$$

and so

$$c^2_0(b) \subset m(E,F).$$

For the second inclusion, let $u = (u_{mn}) \notin \ell^2_\infty$. Then there are increasing sequences (m_i), (n_j) such that

$$|u_{m_i,n_j}| > (ij)^2.$$

Now define the sequence

$$x_{ij} = \begin{cases} \dfrac{1}{ij}, & (i = m_i, j = n_j) \\ 0, & (\text{otherwise}). \end{cases} \tag{3.1}$$

Then, since $x \in c^2_0(b) \subset E$ and

$$u_{ij}x_{ij} = \begin{cases} ij, & (i = m_i, j = n_j) \\ 0, & (\text{otherwise}) \end{cases} \tag{3.2}$$

so $ux \notin \ell^2_\infty$. Therefore $ux \notin F$ by inclusion of $F \subset \ell^2_\infty$; then we have $u \notin m(E,F)$. Hence we have

$$m(E,F) \subset \ell^2_\infty. \quad \square$$

Theorem 3.3. *Let $\mathcal{I}_2$ be a strongly admissible ideal in $2^{\mathbb{N}\times\mathbb{N}}$. Then*
(i) $m\big(F^0_{\mathcal{I}_2}(b)\big) = M\big(F^0_{\mathcal{I}_2}(b)\big) = \ell^2_\infty$.
(ii) $m\big(F_{\mathcal{I}_2}(b)\big) = F_{\mathcal{I}_2}(b)$.

Proof. (i) We show that $m\big(F^0_{\mathcal{I}_2}(b)\big) = \ell^2_\infty$. By Theorem 3.2, the inclusion $m\big(F^0_{\mathcal{I}_2}(b)\big) \subset \ell^2_\infty$ holds.

Now, we show that $\ell_\infty^2 \subset m\big(F_{\mathcal{I}_2}^0(b)\big)$. Let $u \in \ell_\infty^2$ and $z \in F_{\mathcal{I}_2}^0(b)$. Then for $\varepsilon > 0$ we have

$$\{(m,n) \in \mathbb{N}\times\mathbb{N} : |u_{mn}z_{mn}| \geq \varepsilon\} \subseteq \left\{(m,n) \in \mathbb{N}\times\mathbb{N} : |z_{mn}| \geq \frac{\varepsilon}{\|u\|_\infty + 1}\right\}.$$

Since $z \in F_{\mathcal{I}_2}^0(b)$, so we can write

$$\left\{(m,n) \in \mathbb{N}\times\mathbb{N} : |z_{mn}| \geq \frac{\varepsilon}{\|u\|_\infty + 1}\right\} \in \mathcal{I}_2$$

and from property of ideal we have

$$\{(m,n) \in \mathbb{N}\times\mathbb{N} : |u_{mn}z_{mn}| \geq \varepsilon\} \in \mathcal{I}_2.$$

Also since $u, z \in \ell_\infty^2$ so uz is bounded and hence

$$\ell_\infty^2 \subset m\big(F_{\mathcal{I}_2}^0(b)\big).$$

(ii) Let $u \in m\big(F_{\mathcal{I}_2}(b)\big)$. Since $e = (1) \in F_{\mathcal{I}_2}(b)$, then

$$ue = u \in F_{\mathcal{I}_2}(b).$$

Hence we have

$$m\big(F_{\mathcal{I}_2}(b)\big) \subset F_{\mathcal{I}_2}(b).$$

If $u \in F_{\mathcal{I}_2}(b)$, then by Lemma 2.4

$$ux \in F_{\mathcal{I}_2}(b),$$

for each $x \in F_{\mathcal{I}_2}(b)$. This means that $u \in m\big(F_{\mathcal{I}_2}(b)\big)$. Hence we have

$$F_{\mathcal{I}_2}(b) \subset m\big(F_{\mathcal{I}_2}(b)\big). \quad \square$$

Lemma 3.4. $m\big(c_0^2(b)\big) = \ell_\infty^2$.

Proof. Let $x \in c_0^2(b)$ and $\theta \neq u \in \ell_\infty^2$. Then,

$$\|u\|_\infty = \sup_{m,n\in\mathbb{N}} |u_{mn}| < \infty,$$

$$\|x\|_\infty = \sup_{m,n\in\mathbb{N}} |x_{mn}| < \infty$$

and for $\varepsilon > 0$ there exists $N = N(\varepsilon) \in \mathbb{N}$ such that

$$|x_{mn}| < \frac{\varepsilon}{\|u\|_\infty}$$

for every $m, n > N$. Let $z = ux$. Then

$$\begin{aligned} \|z\|_\infty &= \sup_{m,n\in\mathbb{N}} |z_{mn}| \\ &= \sup_{m,n\in\mathbb{N}} |u_{mn}x_{mn}| \\ &\leq \sup_{m,n\in\mathbb{N}} |u_{mn}| \sup_{m,n\in\mathbb{N}} |x_{mn}| < \infty \end{aligned}$$

so z bounded and

$$\begin{aligned} |u_{mn}x_{mn}| &= |u_{mn}\,\|\,x_{mn}| \\ &< \|u\|_\infty \frac{\varepsilon}{\|u\|_\infty} \\ &= \varepsilon \end{aligned}$$

for $m, n > N$. Hence $z \in c_0^2(b)$. Therefore, we have

$$\ell_\infty^2 \subset m\big(c_0^2(b)\big).$$

The inclusion $m\big(c_0^2(b)\big) \subset \ell_\infty^2$ follows from Theorem 3.2. $\square$

Combining Theorem 3.2 and Lemma 3.4, we have the following theorem.

Theorem 3.5. *Let $\mathcal{I}_2$ be a strongly admissible ideal in $2^{\mathbb{N}\times\mathbb{N}}$. Then*

$$m(c_0^2(b), F_{\mathcal{I}_2}^0(b)) = \ell_\infty^2.$$

Theorem 3.6. *Let $\mathcal{I}_2$ be a strongly admissible ideal in $2^{\mathbb{N}\times\mathbb{N}}$. Then*

$$c_0^2(b) \subset m\big(F_{\mathcal{I}_2}(b), c^2(b)\big) \subseteq c^2(b).$$

Proof. For $u \in c_0^2(b)$ and $x \in F_{\mathcal{I}_2}(b) \subset \ell_\infty^2$ by Lemma 3.4 since $ux \in c_0^2(b) \subset c^2(b)$, so we have

$$c_0^2(b) \subset m\big(F_{\mathcal{I}_2}(b), c^2(b)\big).$$

Let $u \in m\big(F_{\mathcal{I}_2}(b), c^2(b)\big)$. Since $e = (1) \in F_{\mathcal{I}_2}(b)$, $ue = u \in c^2(b)$ so we have

$$m\big(F_{\mathcal{I}_2}(b), c^2(b)\big) \subseteq c^2(b). \quad \square$$

Theorem 3.7. *Let $\mathcal{I}_2$ be a strongly admissible ideal in $2^{\mathbb{N}\times\mathbb{N}}$. Then*
(i) *If $c^2(b)$ is a proper subset of $F_{\mathcal{I}_2}(b)$, then $m\big(F_{\mathcal{I}_2}(b), c^2(b)\big) = c_0^2(b)$ and*
(ii) *$m\big(c^2(b), F_{\mathcal{I}_2}(b)\big) = F_{\mathcal{I}_2}(b)$.*

Proof. (i) By Theorem 3.6, we know that

$$c_0^2(b) \subset m\big(F_{\mathcal{I}_2}(b), c^2(b)\big).$$

We show that $u \notin m\big(F_{\mathcal{I}_2}(b), c^2(b)\big)$ for $u \in c^2(b) \setminus c_0^2(b)$. Then there exists a number l such that

$$\lim_{m,n\to\infty} u_{mn} = l \neq 0.$$

Let

$$z \to_{\mathcal{I}_2} 1$$

for $z \in F_{\mathcal{I}_2}(b) \setminus c^2(b)$. Then there is an $\varepsilon > 0$ such that

$$A = \{(m, n) : |z_{mn} - 1| \geq \varepsilon\} \in \mathcal{I}_2.$$

Define $x = (x_{mn})$ by

$$x_{mn} = \chi_{A^c}(m, n)$$

and observe that x is bounded and $\mathcal{I}_2$-convergent to 1; hence $x \in F_{\mathcal{I}_2}(b)$. Also note that xu converges to $\ell \neq 0$ along A^c and to 0 along A; hence $xu \notin c^2(b)$ and thus

$$u \notin m(F_{\mathcal{I}_2}(b), c^2(b)).$$

Hence we have

$$m(F_{\mathcal{I}_2}(b), c^2(b)) \subset c_0^2(b).$$

(ii) Since $e = (1) \in c^2(b)$, we have

$$m(c^2(b), F_{\mathcal{I}_2}(b)) \subseteq F_{\mathcal{I}_2}(b).$$

If $u \in F_{\mathcal{I}_2}(b)$ and if $x \in c^2(b) \subseteq F_{\mathcal{I}_2}(b)$, then ux is bounded and $\mathcal{I}_2$-convergent by Lemma 2.4. Hence we have

$$F_{\mathcal{I}_2}(b) \subset m(c^2(b), F_{\mathcal{I}_2}(b)). \quad \square$$

Theorem 3.8. *Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal with the property (AP2). Then*

$$m(F_{\mathcal{I}_2}^0(b), c_0^2(b)) = \{u \in \ell_\infty^2 : u\chi_E \in c_0^2(b) \text{ for all } E \text{ such that } E \in \mathcal{I}_2\}.$$

Proof. Let $D = \{u \in \ell_\infty^2 : u\chi_E \in c_0^2(b)$ for all E such that $E \in \mathcal{I}_2\}$. First note that if $E \in \mathcal{I}_2$, then

$$\chi_E \in F_{\mathcal{I}_2}^0(b).$$

If $u \in m(F_{\mathcal{I}_2}^0(b), c_0^2(b))$, then

$$u\chi_E \in c_0^2(b).$$

Thus we have

$$m(F_{\mathcal{I}_2}^0(b), c_0^2(b)) \subseteq D.$$

Now let $u \in D$ and $x \in F^0_{\mathcal{I}_2}(b)$. Then by property (AP2) there is an $A \subseteq \mathbb{N} \times \mathbb{N}$ such that

$$x\chi_{A^c} \in c^2_0(b) \quad and \quad A \in \mathcal{I}_2.$$

By property (AP2), as

$$ux = ux\chi_{A^c} + ux\chi_A$$

and both terms of the right hand side are null sequences, $ux \in c^2_0(b)$. Thus we have

$$D \subseteq m(F^0_{\mathcal{I}_2}(b), c^2_0(b)).$$

This completes the proof of theorem. □

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