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The Effect of the Shear Modulus on Planes which is Perpendicular to the Crack's Edge-planes and Parallel to the Crack's Front on the ERR in an Orthotropic Rectangular Prism with a Band Crack

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In this study, a rectangular prism made of an orthotropic material is considered. It is assumed that this prism contains a band crack whose edge-planes are parallel to the upper and lower face planes. It is also assumed that uniformly distributed normal forces are imposed the top and bottom surface of the prism. The aim of this paper is to analyze the effect of the shear modulus on planes which is perpendicular to the crack's edge-planes and parallel to the crack's front on the Energy Release Rate (ERR) for different geometric parameters in a rectangular prism. The mathematical formulation of the corresponding boundary-value problem is carried out within the framework of the 3-dimensional linear theory of elasticity. In order to solve this problem, the 3D finite element method was employed. The numerical results are presented.

Bant Çatlak Içeren Ortotropik Malzemeden Yapılmış bir Dikdörtgen Prizmada, Çatlak Düzlemine Dik ve Çatlak Yüzüne Paralel Kayma Modülünün ERR'ye Etkisi

Anahtar kelimeler
Enerji Salınım Oranı; Bant çatlak, Kayma modülü; 3 Boyutlu Sonlu Elemanlar Yöntemi; Ortotropik Malzeme.

Özet

Bu çalışmada, ortotropik malzemeden yapılmış dikdörtgen prizma ele alınmıştır. Bu prizmanın bir bant çatlak içerdigi ve çatlağın düzlemlerinin, prizmanin alt ve üst düzlemlerine paralel olduğu kabul edilmiştir. Ayrıca prizmanın alt ve üst yüzeylerine düzgün yayılımlı normal kuvvetlerin etki ettiği kabul edilmiştir. Bu çalışmanın amacı; bir dikdörtgen prizmada, çatlak düzlemine dik ve çatlak yüzüne paralel olan kayma modülünün ERR'ye etkisini, farklı geometrik parametreler için incelemektir. Uygun sınır değer problemin matematiksel formülasyonu 3 boyutlu lineer elastistise teorisi çerçevesinde yapılmıştır. Bu problemi çözmek için 3 Boyutlu Sonlu Elemanlar Yöntemi kullanılmıştır. Sayısal sonuçlar sunulmuştur.

1. Introduction

In recent years, the importance of fracture mechanics in engineering applications has increased considerably owing to the growing requirement to predict the behavior of cracked structures under external factors. Therefore, many structural engineers and scientists have concentrated on crack-fracture problems in order to determine the role of the parameters related to the crack's geometry and material properties, the crack's position, method of loading etc. It should be emphasized that Stress Intensity Factor (SIF) and Energy Release Rate (ERR) are critically typical fracture mechanics parameters for this determination. It is known that a wide range of such problems have been studied by many researchers. Approximate and exact solutions deal with ERR and SIF were tabulated in handbooks such as Tada et.al (1985) and Sih (1973).

Moreover, various methods of evaluating the SIF and ERR have been developed so far by Cherepanov (1967), Rice (1968), Shivakumar (1988), Fan et al. (2007), Knowles and Sternberg (1972), Maiti (1992), Gosz et al. (1998) etc. Based on the analyses of the above-mentioned investigations, there are many studies on the effects of the orthotropic and mechanical parameters on the SIF or on the ERR (Akbarov and Turan, 2009, Oneida et al., 2015, Ding and Li, 2014 and Yusufoğlu and Turhan, 2012). However, these studies were discussed within the framework of the two-dimensional (2D) problem formulation. A few of the investigations related to 3D crack problems were carried out by Sukumar et al.(2000) and Li et al.(1998). In these studies, 3D edge crack problem for the rectangular prism was considered. This prism was made of homogeneous, isotropic material.

The present paper considers the 3D corresponding problem for a rectangular prism which contains a band crack. Moreover, it is assumed that the material of the prism is orthotropic. The aim of the present investigation is to determine the influence of the shear modulus in a plane which is perpendicular to the crack's edge-planes and parallel to the crack's front on the values of ERR for various parameters. The 3D finite elements method is utilized so as to provide a solution to the corresponding boundary-value problem.

2. Formulation of the problem

Consider a rectangular prism which contains a band crack at $\{\ell/2-\ell_0/2<x_1<\ell/2+\ell_0/2,$ $x_2=h_\ell, 0\le x_3\le t\}$. Assume that the prism occupies the region $\{0\le x_1\le \ell,\ 0\le x_2\le h, 0\le x_3\le t\}$ in the Cartesian Coordinate system $Ox_1x_2x_3$. The half-prism's geometry (i.e. the case where $0\le x_1\le \ell/2$) is shown in Figure 1.

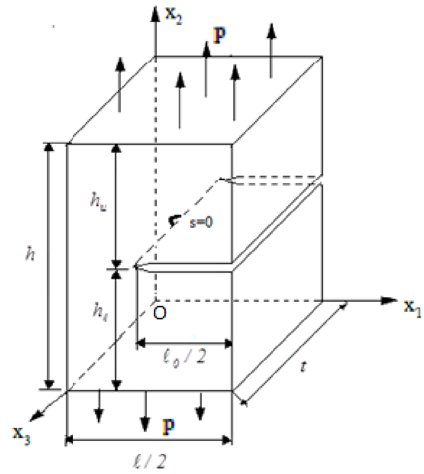

Figure 1. Considered rectangular prism geometry

Suppose that the material of the prism is orthotropic with symmetry axes Ox_1, Ox_2 and Ox_3 The prism is simply supported at its ends and on upper and lower face of the prism act with intensity p uniformly distributed normal forces (Figure 1). To find ERR at the band crack front is investigated stress-state in the prism. Within the framework 3D linear theory of elasticity for anisotropic bodies, the stress-state can be determined by the solution to the boundary-value problem given below:

$$\frac{\partial\sigma_{1i}}{\partial x_1}+\frac{\partial\sigma_{2i}}{\partial x_2}+\frac{\partial\sigma_{3i}}{\partial x_3}=0,$$

$$\sigma_{ii}=A_{i1}\varepsilon_{11}+A_{i2}\varepsilon_{22}+A_{i3}\varepsilon_{33}, A_{ij}=A_{ji},$$

(1)

$$\sigma_{ij}=2\mu_{ij}\varepsilon_{ij}, \text{ at } i\neq j$$

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right), \qquad i,j=1,2,3$$

Boundary conditions:

$$u_2\big|_{x_1=0}=u_2\big|_{x_1=\ell}=0,\ u_2\big|_{x_3=0}=u_2\big|_{x_3=t}=0,$$

$$\sigma_{2i}\big|_{x_2=0}=\sigma_{2i}\big|_{x_2=h}=p\delta_i^2,\ \ \sigma_{1i}\big|_{x_1=0}=\sigma_{1i}\big|_{x_1=\ell}=0,$$

$$\sigma_{3i}\big|_{x_3=0}=\sigma_{3i}\big|_{x_3=t}=0 \qquad (2)$$

$$\sigma_{2i}\Big|_{\substack{x_2=h_{\ell\pm0} \\ (\ell/2-\ell_0/2)<x_1<(\ell/2+\ell_0/2) \\ 0<x_3<t}} = 0\,,\ i=1,2,3\,. \qquad (3)$$

where

$$A_{ij}=(-1)^{i+j}a_{ij}\times(\det\|a_{nm}\|)^{-1},\ a_{11}=\frac{1}{E_1},\quad a_{12}=-\frac{\nu_{12}}{E_2}$$

$$a_{13}=-\frac{\nu_{13}}{E_3},\quad a_{23}=-\frac{\nu_{23}}{E_3},\quad a_{22}=\frac{1}{E_2},\quad a_{33}=\frac{1}{E_3},$$

$$a_{ij}=a_{ji}\,,\ G_{ij}=\mu_{ij}\,,\ i\neq j\,,\ i,j=1,2,3\ . \qquad (4)$$

In Eq. (4), E_1, E_2 and E_3 denote the moduli of elasticity in the Ox_1, Ox_2 and Ox_3 directions, respectively; ν_{12} , ν_{23} and ν_{13} are Poisson's ratios of the material; and G_{12}, G_{23} and G_{13} are the shear moduli of the material in the Ox_1x_2, Ox_2x_3 and Ox_1x_3 planes, respectively. (Lekhnitskii , 1981). Thus, the mathematical formulation of the problem considered is complete.

3. FEM Modelling

For the 3D FEM modeling of the boundary-value problem, the standard Ritz technique (Zienkiewicz and Taylor, 1989) and the following functional is used:

$$\Pi=\frac{1}{2}\iiint\limits_{\Omega-\Omega'}(\sigma_{11}\varepsilon_{11}+2\sigma_{12}\varepsilon_{12}+2\sigma_{13}\varepsilon_{13}$$
$$+2\sigma_{23}\varepsilon_{23}+\sigma_{22}\varepsilon_{22}+\sigma_{33}\varepsilon_{33})d\Omega$$
$$-\int_0^t\int_0^\ell pu_2\Big|_{x_2=h}dx_1dx_3-\int_0^t\int_0^\ell pu_2\Big|_{x_2=0}dx_1dx_3 \qquad (5)$$

where

$$\Omega=\{0\le x_1\le\ell,\ 0\le x_2\le h,\ 0\le x_3\le t\},$$
$$\Omega'=\{(\ell-\ell_0)/2<x_1<(\ell+\ell_0)/2,\ x_2=h_\ell-0,\ 0\le x_3\le t\}\cup \qquad (6)$$
$$\{(\ell-\ell_0)/2<x_1<(\ell+\ell_0)/2,\ x_2=h_\ell+0,\ 0\le x_3\le t\ \}$$

In this case, from the first variation of functional (5) with respect to the displacement, i.e. from the equation $\delta\Pi_u=0$, the equilibrium equation of (1) and the boundary conditions with respect to the forces in Eq. (2) are found. In such a way, the validity of functional (5) in the 3D FEM modeling of boundary value problems is ensured.

In the Finite Element solution by using the symmetry of $x_1=\ell/2$, only half-region Ω of the prism considered (Figure 1). The half part of the prism is discretized into eight-node rectangular brick finite elements (Figure 2), i. e. the region Ω is expressed as $\Omega=\bigcup_{k=1}^{M}\Omega_k$ where Ω_k is a region of the $k-$th finite element. Using a normalized local coordinate system $(O\xi\eta\zeta)$ the shape functions of the brick elements are written as,

$$N_1(\xi,\eta,\zeta)=\frac{1}{8}(1\ \eta)(1\ \zeta)\ 1-\ ,\ N_2(\xi,\eta,\zeta)=\frac{1}{8}(1\ \eta)(1\ \zeta)\ 1-\ ,$$
$$N_3(\xi,\eta,\zeta)=\frac{1}{8}(1\ \eta)(1\ \zeta)\ 1+\ ,\ N_4(\xi,\eta,\zeta)=\frac{1}{8}(1\ \eta)(1\ \zeta)\ 1+\ ,$$
$$N_5(\xi,\eta,\zeta)=\frac{1}{8}(1\ \eta)(1\ \zeta)\ 1-\ ,\ N_6(\xi,\eta,\zeta)=\frac{1}{8}(1\ \eta)(1\ \zeta)\ 1-\ ,$$
$$N_7(\xi,\eta,\zeta)=\frac{1}{8}(1\ \eta)(1\ \zeta)\ 1+\ ,\ N_8(\xi,\eta,\zeta)=\frac{1}{8}(1\ \eta)(1\ \zeta)\ 1+ \qquad (7)$$

(Zienkiewicz and Taylor, 1989). The transformation relation between the $Ox_1x_2x_3$ and $O\xi\eta\zeta$ coordinate systems is expressed as,

$$\xi=\frac{x_1-x_{10}}{\beta},\eta=\frac{x_2-x_{20}}{\alpha},\zeta=\frac{x_3-x_{30}}{\wp}\,. \qquad (8)$$

In Eq. (8), x_{10}; x_{20}; x_{30} are components of the vector $\overrightarrow{OO'}$ at $Ox_1x_2x_3$ coordinate system.

In this study, it is used the displacement-based FEM, in other words, according to FEM procedure, only displacements at the nodes are supposed to be unknown. So, the displacement functions are defined as follows:

$$u^{(k)}\approx N^{(k)}a^{(k)}\ ,k=1,2,.....M \qquad (9)$$

where

$$(a^{(k)})^T = \left\{u_{11}^k\ u_{21}^k\ u_{31}^k\ u_{12}^k\ u_{22}^k\ u_{32}^k \ldots u_{18}^k\ u_{28}^k\ u_{38}^k\right\}$$

$$(N^{(k)})^T = \begin{bmatrix} N_1^k & 0 & 0 & N_2^k & 0 & 0 & \ldots & N_8^k & 0 & 0 \\ 0 & N_1^k & 0 & 0 & N_2^k & 0 & \ldots & 0 & N_8^k & 0 \\ 0 & 0 & N_1^k & 0 & 0 & N_2^k & \ldots & 0 & 0 & N_8^k \end{bmatrix}$$

$$(u^{(k)})^T = \left\{u_1^k(x_1,x_2,x_3)\ u_2^k(x_1,x_2,x_3)\ u_3^k(x_1,x_2,x_3)\right\} \tag{10}$$

After some mathematical manipulations, finally yields the following system of algebraic equations :

$$\mathbf{K}\ \mathbf{a} = \mathbf{r} \tag{11}$$

where, **K** is the stiffness matrix, **a** is the displacement vector at finite-element nodes and **r** is the force vector.

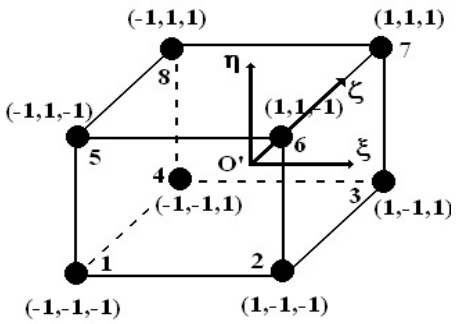

Figure 2. The geometry of brick (rectangular) finite element

After consideration of the FEM modelling, the ERR (denoted by γ) is calculated by using the expression:

$$\gamma \approx \frac{U(S_c + \Delta S(s/t)) - U(S_c)}{\Delta S(s/t)} \tag{12}$$

where

$$U(S_c) = \frac{1}{2}\iiint\limits_{\Omega-\Omega'} \left(\sigma_{11}\varepsilon_{11} + 2\sigma_{12}\varepsilon_{12} + 2\sigma_{13}\varepsilon_{13} + 2\sigma_{23}\varepsilon_{23} + \sigma_{22}\varepsilon_{22} + \sigma_{33}\varepsilon_{33}\right)d\Omega \tag{13}$$

$$U(S_c + \Delta S(s/t)) = \frac{1}{2}\iiint\limits_{\Omega-\Omega''} \left(\sigma_{11}\varepsilon_{11} + 2\sigma_{12}\varepsilon_{12} + 2\sigma_{13}\varepsilon_{13} + 2\sigma_{23}\varepsilon_{23} + \sigma_{22}\varepsilon_{22} + \sigma_{33}\varepsilon_{33}\right)d\Omega \tag{14}$$

In the above expressions, U denotes the strain energy, S_c is the area of the crack's edge surface.

First, we calculate the strain energy for the unperturbed case using (13). Then, we apply a small perturbation (area of which is defined by $\Delta S(s/t)$ which is itself determined by the parameter s/t on the crack front) on the area of the crack surface and calculate the strain energy again for the perturbed case using (14). It should be noted that the arc length co-ordinate s is measured along the crack front from the center of the crack ($s=0$) to the point where it reaches the free surface ($s=t/2$). The domains Ω and Ω' in (12) are determined by the expressions in (6), $\Omega'' = (\Omega' + \Omega_{\Delta S(s/t)})$ and $\Omega_{\Delta S(s/t)}$ are the domains corresponding to the aforementioned perturbation of the crack's edge area.

In the calculation procedure, the $\Delta S(s/t)$ values are chosen small enough to ensure the numerical convergence. The values of γ for various values of $\Delta S(s/t)$ are calculated. Obviously, the results are improved with smaller $\Delta S(s/t)$. Moreover, dimensionless Energy Release Rate (denoted by ERR) is used and this parameter is defined by $\mathrm{ERR} = \frac{\gamma}{E_1}$.

4. Numerical Results

Before obtaining numerical results, the PC programs composed and used by the author are tested on the problems considered in Sukumar et al. (2000) and Li et al. (1998). To allow comparison with corresponding numerical results Sukumar et al. (2000) and Li et al. (1998), first, it is considered a rectangular prism which contains a single edge crack. The top and bottom surface of this prism impose uniformly distributed normal forces as shown Figure 3. The geometrical parameters are taken as $\ell_0/\ell = 0.5$, $t/\ell = 0.75$ and $h/\ell = 0.875$.

In Figure 4, the values of the SIF for the mode I for isotropic case i.e. $E_1 = E_2 = E_3 = E$, $\nu_{12} = \nu_{13} = \nu_{23} = \nu$, $G_{12} = E/(2(1+\nu))$ and $\nu = 0.3$ are presented and comparisons with the results obtained by Sukumar et al. (2000) and Li et al. (1998) are shown. Here, two different finite meshes are considered in the FEM model a) Mesh 1 consists of 20x20x20 brick elements and b) Mesh 2 consists of 24x24x24 brick elements along the Ox_1, Ox_2 and Ox_3 axes, respectively. As seen, in Figure 4 , there is close agreement between the results obtained and the reference solutions. Thus, it is verified the validity of the present FEM modelling and PC programs.

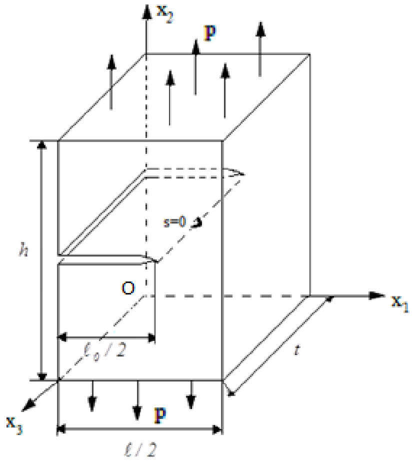

Figure 3. Edge cracked rectangular prism

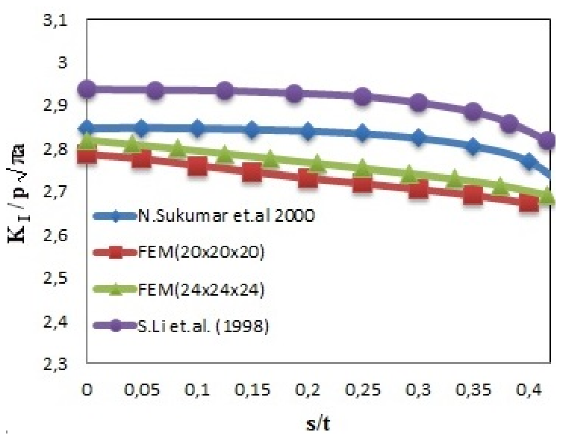

Figure 4. Comparison of the values of K_I with the present study and the reference papers for the edge crack problem in isotropic case.

Now, let us focus on the band crack problem regarding to the effect of the shear modulus in a plane which is perpendicular to the crack's edge-planes and parallel to the crack's front (i.e. in the present case the modulus G_{23}) on the values of the ERR for various parameter (Figure 1). Assume that the material of the prism is orthotropic and the numerical investigations are made for $\nu_{12} = \nu_{13} = \nu_{23} = 0.3$, $G_{12}/E_1 = G_{13}/E_1 = 0.09$, $E_2/E_1 = E_3/E_1 = 0.5$, $\ell_0/\ell = 0.5$, $t/\ell = 0.75$ and $h/\ell = 0.875$. The half part of the prism is discretized into brick elements with eight nodes, where the Ox_1 direction is taken as 20, the Ox_2 direction is taken as 20, and the Ox_3 direction is taken as 20. The results are presented below.

The graph of the dependencies between the ERR and s/t is given in Figure 5 . As can be seen in this graph that the increase of the absolute values of the ratio s/t causes the decrease of the values of the ERR. Moreover, in Figure 5, ERR reaches maximum value (denoted by the symbol (★)) at the center of crack (at $s/t = 0$). This result again confirms the trustiness of the algorithm and PC programs composed by the author.

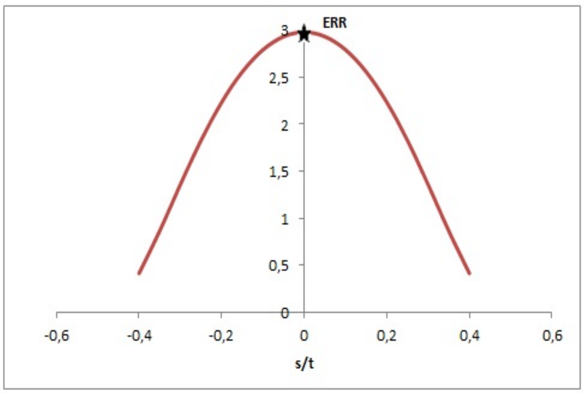

Figure 5. The graphs of the dependencies between ERR and s/t at $\ell_0/2\ell = 0.25$ and $G_{23}/E_1 = 0.09$

In Table 1, the effect of the ratios G_{23}/E_1 (where G_{23} is the shear modulus in the Ox_2x_3 plane) on the values of the ERR for various values of s/t and $\ell_0/2\ell$ are given for the case where $h_\ell = h_u = h/2$. It is seen that the values of the ERR significantly increase with decreasing of the ratio of G_{23}/E_1. As expected, this increase becomes more pronounced with crack length, i. e., with $\ell_0/2\ell$.

Table 1. Effect of G_{23}/E_1 and $\ell_0/2\ell$ on the ERR for various s/t at $h_\ell = h_u = h/2$.

$\ell_0/2\ell$	s/t	G_{23}/E_1		
		0.09	**0.06**	**0.03**
0.25	0	2.9810	5.1913	10.7757
	0.05	2.9339	5.1310	10.7416
	0.10	2.7930	4.9470	10.6249
	0.15	2.5588	4.6350	10.3799
	0.20	2.2345	4.1683	9.9269
	0.25	1.8280	3.5475	9.1470
	0.30	1.3587	2.7659	7.8826
	0.35	0.8653	1.8570	5.9731
	0.40	0.4142	0.9333	3.4287
0.30	0	3.2233	6.0063	14.2036
	0.05	3.1701	5.9272	14.1108
	0.10	3.0111	5.6882	13.8175
	0.15	2.7490	5.2846	13.2799
	0.20	2.3897	4.7108	12.4246
	0.25	1.9451	3.9640	11.1504
	0.30	1.4388	3.0550	9.3359
	0.35	0.9133	2.0314	6.8799
	0.40	0.4378	1.0175	3.8692

Table 2 shows the influence of the crack's location on ERR various values of G_{23}/E_1. The parameter h_u/ℓ shows the thickness of the part of the prism at the top of the crack. As seen, the values of the ERR increase with the crack moving closer to the upper face plane of the prism. This result agrees with the well-known mechanical considerations. Moreover, it also follows from this table that this effect slightly increases with decreasing of the ratio G_{23}/E_1.

Consider now the influence of the parameter t/ℓ on the ERR for various t/ℓ, G_{23}/E_1 and s/t at $h_\ell = h_u = h/2$ and $\ell_0/2\ell = 0.25$; where t/ℓ is the length of the prism along the Ox_3 axis. These results are given in Table 3. It is seen that the values of the ERR increase with an increase of the parameter t/ℓ. A decrease of the ratio of the G_{23}/E_1 causes a decrease in the influence of the parameter t/ℓ on the ERR.

Table 3. Effect of G_{23}/E_1 and t/ℓ on the ERR for various s/t at $\ell_0/2\ell = 0.25$ and $h_\ell = h_u = h/2$.

G_{23}/E_1	s/t	t/ℓ	
		0.75	**1**
0.09	0	2.9810	4.6663
	0.05	2.9339	4.6093
	0.10	2.7930	4.4361
	0.15	2.5588	4.1410
	0.20	2.2345	3.7158
	0.25	1.8280	3.1535
	0.30	1.3587	2.4566
	0.35	0.8653	1.6563
	0.40	0.4142	0.8463
0.06	0	5.1913	6.4221
	0.05	5.1310	6.3786
	0.10	4.9470	6.2414
	0.15	4.6350	5.9902
	0.20	4.1683	5.5905
	0.25	3.5475	4.9947
	0.30	2.7659	4.1477
	0.35	1.8570	3.0144
	0.40	0.9333	1.6651

Table 2. The effect of h_u/ℓ and G_{23}/E_1 on the ERR for various values of s/t at $\ell_0/2\ell = 0.25$.

s/t	G_{23}/E_1							
	0.09				**0.03**			
	h_u/ℓ				h_u/ℓ			
	0.4375	**0.30625**	**0.2625**	**0.175**	**0.4375**	**0.30625**	**0.2625**	**0.175**
0	2.9810	3.4845	3.9792	6.2703	10.7757	11.6841	12.4756	15.4977
0.05	2.9339	3.4307	3.9188	6.1787	10.7416	11.6535	12.4488	15.4881
0.10	2.7930	3.2698	3.7381	5.9045	10.6249	11.5452	12.3502	15.4346
0.15	2.5588	3.0029	3.4360	5.4500	10.3799	11.3078	12.1234	15.2612
0.20	2.2345	2.6334	3.0267	4.8216	9.9269	10.8515	11.6692	14.8338

0.25	1.8280	2.1702	2.5038	4.0336	9.1470	10.0438	10.8426	13.9560
0.30	1.3587	1.6324	1.8986	3.1133	7.8826	8.7142	9.4588	12.3777
0.35	0.8653	1.0582	1.2468	2.1097	5.9731	6.6928	7.3366	9.8543
0.40	0.4142	0.5177	0.6211	1.1091	3.4287	3.9526	4.4238	6.2664

As seen from the tables, the effect of the problem parameters on the ERR is more considerable at the center of the crack $(s = 0)$ than at the points that are close to the free face planes.

5. Conclusions

Thus, in the present paper we deal with rectangular orthotropic prism under the action of the uniformly distributed normal forces on the upper and lower face planes with a band crack. The investigation is focused on the effect shear modulus in a plane which is perpendicular to the crack's edge-planes and parallel to the crack's front (i. e. the modulus G_{23}) and geometrical parameters on the ERR. By employing the three-dimensional FEM modeling, the corresponding boundary-value problem is solved. The mathematical formulation of the corresponding boundary value problems is presented within the scope of the three-dimensional linear theory of elasticity for anisotropic bodies. Based on these analyses, the following concrete conclusions can be drawn:

- The values of the ERR significantly increase with decreasing of the shear modulus in a plane which is perpendicular to the crack's edge-planes and parallel to the crack's front (i. e. the modulus G_{23}). This increase is more influenced with crack length;

- The values of the ERR increase as the crack approaches the free upper face of the prism. This increase is larger with decreasing G_{23} ;

-The effect of the problem parameters to the ERR at the center of the crack, i.e. at $s / t = 0$ is more notable than at the points that are close to the free face surface of the prism; and

- The values of the ERR increase with increasing of the length of the prism along the Ox_3 axis.

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