Multipliers for bounded convergent double sequences
Erdinç Dündar and Yurdal Sever

Citation: AIP Conference Proceedings 1611, 344 (2014); doi: 10.1063/1.4893858
View online: http://dx.doi.org/10.1063/1.4893858
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1611?ver=pdfcov
Published by the AIP Publishing

Articles you may be interested in
On some new double spaces of -convergent and -bounded sequences defined by Orlicz function
AIP Conf. Proc. 1558, 785 (2013); 10.1063/1.4825611

Rough statistically convergent double sequences

Convergence of correlations in multiply scattering media

-function converging sequences
Am. J. Phys. 70, 180 (2002); 10.1119/1.1427087

A bounded convergence theorem for the Feynman integral
Multipliers for bounded convergent double sequences

Erdinç Dündar and Yurdal Sever

Department of Mathematics, Afyon Kocatepe University, 03200 Afyonkarahisar, Turkey

Abstract. In this paper, we investigate multipliers for bounded convergence of double sequences and study some properties and relations between \( \ell_2^\infty \), \( c^2(b) \) and \( c^2_0(b) \).

Keywords: Double Sequences, Multiplier

PACS: 02.30.Lt

INTRODUCTION

Hill [8] was the first who applied methods of functional analysis to double sequences. Also, Kull [10] applied methods of functional analysis of matrix maps of double sequences. A lot of useful developments of double sequences in summability methods can be seen in [1, 9, 12, 15].

The study of the multipliers of one sequence space into another is a well-established area of research and has been the object of several investigations over the last fifty years. Demirci and Orhan [3] studied the bounded multiplier space of all bounded \( A \)-statistically convergent sequences, and using the “\( \beta N \) program” they gave an analogue of a result of Fridy and Miller [6] for bounded multipliers. Connor, Demirci and Orhan [2] studied multipliers and factorizations for bounded statistically convergent sequences and a related result. Dündar and Sever [5] studied multipliers for bounded statistical convergence of double sequences in \( \mu_2 \)-density. Yardımcı [16] studied multipliers for bounded \( I \)-convergent sequences. Also, Dündar and Altay [4] investigated analogous results of multipliers for bounded \( I_2 \)-convergent double sequences.

In this paper, we investigate multipliers for bounded convergence of double sequences and study some properties and relations between \( \ell_2^\infty \), \( c^2(b) \) and \( c^2_0(b) \).

DEFINITIONS AND NOTATIONS

Throughout the paper, \( \mathbb{N} \) denotes the set of all positive integers while \( \mathbb{R} \) represents the set of all real numbers.

Now, we recall the concepts of double sequence, Pringsheim’s convergence, multiplier for bounded convergence of the double sequences [1, 4, 7, 8, 11, 13, 14].

A double sequence \( x = (x_{mn})_{m,n \in \mathbb{N}} \) of real numbers is said to be convergent to \( L \in \mathbb{R} \) if for any \( \varepsilon > 0 \), there exists \( N_\varepsilon \in \mathbb{N} \) such that

\[
|x_{mn} - L| < \varepsilon,
\]

whenever \( m, n > N_\varepsilon \). In this case we write

\[
\lim_{m,n \to \infty} x_{mn} = L.
\]

A double sequence \( x = (x_{mn})_{m,n \in \mathbb{N}} \) of real numbers is said to be bounded if there exists a positive real number \( M \) such that

\[
|x_{mn}| < M,
\]

for all \( m, n \in \mathbb{N} \), that is

\[
\|x\|_\infty = \sup_{m,n} |x_{mn}| < \infty.
\]

Note that in contrast to the case for single sequences, a convergent double sequence need not be bounded. By \( \ell_2^\infty \), \( c^2(b) \) and \( c^2_0(b) \), we denote the spaces of all bounded, bounded convergent and bounded null double sequences, respectively.
Let $E$ and $F$ be two double sequence spaces. A multiplier from $E$ into $F$ is a sequence $u = (u_{mn})_{m,n \in \mathbb{N}}$ such that

$$ux = (u_{mn}x_{mn}) \in F,$$

whenever $x = (x_{mn})_{m,n \in \mathbb{N}} \in E$. The linear space of all such multipliers will be denoted by $m(E,F)$.

If $E = F$, then we write $m(E)$ instead of $m(E,F)$.

Now we begin with quoting the lemmas due to Dündar and Altay [4] which are needed throughout the paper.

**Lemma 1.** [4, Theorem 3.2] If $E$ and $F$ are subspaces of $\ell_2^\omega$ that contain $c_0^2(b)$, then $c_0^2(b) \subset m(E,F) \subset \ell_2^\omega$.

**Lemma 2.** [4, Lemma 3.4] $m(c_0^2(b)) = \ell_2^\omega$.

**MAIN RESULTS**

In this section, we deal with the multipliers on or into $\ell_2^\omega, c_0^2(b)$ and $c_0^2(b)$.

**Theorem 3.** $m(\ell_2^\omega) = \ell_2^\omega$.

**Proof.** Let $u = (u_{mn}), x = (x_{mn}) \in \ell_2^\omega$. Then, we have

$$\|u\|_\omega = \sup_{m,n} |u_{mn}| < \infty,$$

$$\|x\|_\omega = \sup_{m,n} |x_{mn}| < \infty.$$

Now, let $z = ux$. Then, we have

$$\|z\|_\omega = \sup_{m,n} |z_{mn}| = \sup_{m,n} |u_{mn}x_{mn}| \leq \sup_{m,n} |u_{mn}| \sup_{m,n} |x_{mn}| < \infty$$

and so $u \in m(\ell_2^\omega)$. This implies that $\ell_2^\omega \subset m(\ell_2^\omega)$.

Conversely, since $e \in \ell_2^\omega$ ($e$ is the sequence of all 1’s), we have

$$m(\ell_2^\omega) \subset \ell_2^\omega.$$

This completes the proof of the theorem. \qed

**Theorem 4.** $m(\ell_2^\omega, c_0^2(b)) = c_0^2(b)$.

**Proof.** Let $u \in c_0^2(b)$ and $\theta \neq x \in \ell_2^\omega$. Then, we have

$$\|x\|_\infty = \sup_{m,n \in \mathbb{N}} |x_{mn}| < \infty,$$

$$\|u\|_\infty = \sup_{m,n \in \mathbb{N}} |u_{mn}| < \infty.$$

and for $\varepsilon > 0$ there exists $N = N(\varepsilon) \in \mathbb{N}$ such that

$$|u_{mn}| < \frac{\varepsilon}{\|x\|_\infty}$$

for every $m, n > N$. Let $z = xu$. Then, we have

$$\|z\|_\omega = \sup_{m,n \in \mathbb{N}} |z_{mn}| = \sup_{m,n \in \mathbb{N}} |x_{mn}u_{mn}| \leq \sup_{m,n \in \mathbb{N}} |x_{mn}| \sup_{m,n \in \mathbb{N}} |u_{mn}| < \infty,$$
so \( z \) is bounded and
\[
|x_{mn}u_{mn}| = |x_{mn}|u_{mn} < \|x\|_\infty \frac{\varepsilon}{\|x\|_\infty} = \varepsilon
\]
for \( m,n > N \). Hence, we have \( z \in c_0^2(b) \). This shows that
\[
c_0^2(b) \subset m\left(\ell_\infty^2, c_0^2(b)\right).
\]
Now, since \( e \in \ell_\infty^2 \) we have
\[
m\left(\ell_\infty^2, c_0^2(b)\right) \subset c_0^2(b).
\]
This completes the proof of the theorem. \( \square \)

**Theorem 5.** \( m(c_0^2(b), \ell_\infty^2) = \ell_\infty^2 \).

**Proof.** Since \( c_0^2(b) \subset \ell_\infty^2 \) then by Theorem 3 we have
\[
m(c_0^2(b), \ell_\infty^2) \subset \ell_\infty^2.
\]
Now, let \( u \in \ell_\infty^2 \) and \( x \in c_0^2(b) \). Then, it is clear that
\[
ux \in \ell_\infty^2
\]
and so
\[
\ell_\infty^2 \subset m(c_0^2(b), \ell_\infty^2).
\]
Hence, we have \( m(c_0^2(b), \ell_\infty^2) = \ell_\infty^2 \). \( \square \)

**Theorem 6.** \( m(c^2(b), \ell_\infty^2) = \ell_\infty^2 \).

**Proof.** Since \( c^2(b) \subset \ell_\infty^2 \) then by Theorem 3 we have
\[
m(c^2(b), \ell_\infty^2) \subset \ell_\infty^2.
\]
Now, let \( u \in \ell_\infty^2 \) and \( x \in c^2(b) \subset \ell_\infty^2 \). Then, we have
\[
ux \in \ell_\infty^2
\]
and so
\[
\ell_\infty^2 \subset m(c^2(b), \ell_\infty^2).
\]
This completes the proof of the theorem. \( \square \)

**Theorem 7.** \( m(c^2(b)) = c^2(b) \).

**Proof.** Let \( e = (1) \in c^2(b) \). Then, we have
\[
ue = u \in c^2(b)
\]
for each \( u \in m(c^2(b)) \) and so
\[
m(c^2(b)) \subset c^2(b).
\]
Now, let \( u \notin c^2(b) \). Since \( e \in c^2(b) \), then we have
\[
ue = u \notin c^2(b)
\]
so \( c^2(b) \subset m(c^2(b)) \). \( \square \)

**Theorem 8.** \( m(c^2(b), c_0^2(b)) = c_0^2(b) \).
Proof. Let \( u \in c_0^2(b) \) and \( e \in c^2(b) \). Then, we have

\[
ue = u \in c_0^2(b)
\]

and so

\[
c_0^2(b) \subset m(c^2(b), c_0^2(b)).
\]

Let \( u \not\in c_0^2(b) \). Since \( e \in c^2(b) \) then,

\[
ue = u \not\in c_0^2(b)
\]

and so

\[
u \not\in m(c^2(b), c_0^2(b)).
\]

Hence, we have

\[
m(c^2(b), c_0^2(b)) \subset c_0^2(b).
\]

This completes the proof of the theorem.

\[\square\]

**Theorem 9.** \( m(c_0^2(b), c^2(b)) = \ell^2_w \).

Proof. Since \( c_0^2(b) \subset \ell^2_w \) and \( c^2(b) \subset \ell^2_w \), by Lemma 1

\[
m(c_0^2(b), c^2(b)) \subset \ell^2_w.
\]

Conversely, since \( c_0^2(b) \subset c^2(b) \), by Lemma 2

\[
\ell^2_w \subset m(c_0^2(b), c^2(b)).
\]

Therefore, we have

\[
m(c_0^2(b), c^2(b)) = \ell^2_w.
\]

\[\square\]

**ACKNOWLEDGMENTS**

The authors would like to thank to Prof. Bilâl Altay (Inönü University, Turkey) for his contributions and the constructive comments which improved the presentation of the paper.

**REFERENCES**