Filomat 28:5 (2015), 907–915
DOI 10.2298/FIL1405907S

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Regularly Ideal Convergence and Regularly Ideal Cauchy Double Sequences in 2-Normed Spaces

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Abstract. In this paper, we introduce the notions of $(\mathcal{I}_2,\mathcal{I})$, $(\mathcal{I}_2^*,\mathcal{I}^*)$-convergence and $(\mathcal{I}_2,\mathcal{I})$, $(\mathcal{I}_2^*,\mathcal{I}^*)$-Cauchy double sequence in regular sense in 2-normed spaces. Also, we study some properties of these concepts.

1. Introduction, Notations and Definitions

Throughout the paper $\mathbb{N}$ and $\mathbb{R}$ denote the set of all positive integers and the set of all real numbers, respectively. The concept of convergence of a sequence of real numbers has been extended to statistical convergence independently by Fast [6] and Schoenberg [26]. This concept was extended to the double sequences by Mursaleen and Edely [17]. The idea of $\mathcal{I}$-convergence was introduced by Kostyrko et al. [15] as a generalization of statistical convergence which is based on the structure of the ideal $\mathcal{I}$ of subset of the set of natural numbers [6, 7]. Nuray and Ruckle [21] independently introduced the same with another name generalized statistical convergence. Das et al. [2] introduced the concept of $\mathcal{I}_2$-convergence of double sequences in a metric space and studied some properties. Dündar and Altay [4] studied the concepts of $\mathcal{I}_2$-Cauchy and $\mathcal{I}_2^*$-Cauchy for double sequences and they gave the relation between $\mathcal{I}_2$ and $\mathcal{I}_2^*$-convergence of double sequences of functions defined between linear metric spaces. A lot of development have been made in this area after the works of [3, 16, 18–20, 25, 27–29].

The concept of 2-normed spaces was initially introduced by Gähler [8, 9] in the 1960's. Since then, this concept has been studied by many authors, see for instance [10–12, 14]. Şahiner et al. [27] and Gürdal [14] studied $\mathcal{I}$-convergence in 2-normed spaces. Gürdal and Açık [13] investigated $\mathcal{I}$-Cauchy and $\mathcal{I}^*$-Cauchy sequences in 2-normed spaces. Sarabadan et al. [23, 24] investigated $\mathcal{I}_2$ and $\mathcal{I}_2^*$-convergence of double sequences in 2-normed spaces. They also examined the concepts $\mathcal{I}_2$-limit points and $\mathcal{I}_2$-cluster points in 2-normed spaces. Dündar and Sever [5] introduced the notions of $\mathcal{I}_2$ and $\mathcal{I}_2^*$-Cauchy double sequences, and studied their some properties with $(AP2)$ in 2-normed spaces.

In this paper, we introduce the notions of $(\mathcal{I}_2,\mathcal{I})$, $(\mathcal{I}_2^*,\mathcal{I}^*)$-convergence and $(\mathcal{I}_2,\mathcal{I})$, $(\mathcal{I}_2^*,\mathcal{I}^*)$-Cauchy double sequence in regular sense in 2-normed spaces. Also, we study some properties of these concepts.

Now, we recall the concept of ideal, ideal convergence of sequences, double sequences, 2-normed space and some fundamental definitions and notations (See [1, 2, 8, 11, 13, 15, 22–24]).

2010 *Mathematics Subject Classification*. Primary 40A35, 40B05; Secondary 46A70
Keywords. Ideal; Double Sequences; $\mathcal{I}_2$-Convergence; $\mathcal{I}_2$-Cauchy; 2-normed spaces
Received: 24 September 2013; Revised 21 January 2014; Accepted: 30 January 2014
Communicated by Eberhard Malkowsky
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A double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ of real numbers is said to be *convergent* to $L \in \mathbb{R}$ in Pringsheim's sense, if for any $\varepsilon > 0$ there exists $N_\varepsilon \in \mathbb{N}$ such that $|x_{mn} - L| < \varepsilon$ whenever $m, n > N_\varepsilon$. In this case we write $P - \lim_{m,n\to\infty} x_{mn} = L$ or $\lim_{m,n\to\infty} x_{mn} = L$.

Let $X \neq \emptyset$. A class $\mathcal{I}$ of subsets of X is said to be an *ideal* in X provided:

(i) $\emptyset \in \mathcal{I}$,

(ii) $A, B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$,

(iii) $A \in \mathcal{I}, B \subset A$ implies $B \in \mathcal{I}$.

$\mathcal{I}$ is called a *nontrivial ideal* if $X \notin \mathcal{I}$.

Let $X \neq \emptyset$. A non empty class $\mathcal{F}$ of subsets of X is said to be a *filter* in X provided:

(i) $\emptyset \notin \mathcal{F}$,

(ii) $A, B \in \mathcal{F}$ implies $A \cap B \in \mathcal{F}$,

(iii) $A \in \mathcal{F}, A \subset B$ implies $B \in \mathcal{F}$.

If $\mathcal{I}$ is a nontrivial ideal in X, $X \neq \emptyset$, then the class

$$\mathcal{F}(\mathcal{I}) = \{M \subset X : (\exists A \in \mathcal{I})(M = X \backslash A)\}$$

is a filter on X, called the filter associated with $\mathcal{I}$.

A nontrivial ideal $\mathcal{I}$ in X is called *admissible* if $\{x\} \in \mathcal{I}$ for each $x \in X$.

Throughout the paper we take $\mathcal{I}$ as a nontrivial admissible ideal in $\mathbb{N}$.

Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be a nontrivial ideal and (X, ρ) be a metric space. A sequence (x_n) of elements of X is said to be $\mathcal{I}$-convergent to $L \in X$, if for each $\varepsilon > 0$ we have $A(\varepsilon) = \{n \in \mathbb{N} : \rho(x_n, L) \geq \varepsilon\} \in \mathcal{I}$.

Throughout the paper we take $\mathcal{I}_2$ as a nontrivial admissible ideal in $\mathbb{N} \times \mathbb{N}$.

A nontrivial ideal $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ is called strongly admissible if $\{i\} \times \mathbb{N}$ and $\mathbb{N} \times \{i\}$ belong to $\mathcal{I}_2$ for each $i \in \mathbb{N}$.

It is evident that a strongly admissible ideal is also admissible.

Let $\mathcal{I}_2^0 = \{A \subset \mathbb{N}\times\mathbb{N} : (\exists m(A) \in \mathbb{N})(i, j \geq m(A) \Rightarrow (i, j) \notin A)\}$. Then $\mathcal{I}_2^0$ is a nontrivial strongly admissible ideal and clearly an ideal $\mathcal{I}_2$ is strongly admissible if and only if $\mathcal{I}_2^0 \subset \mathcal{I}_2$.

Let (X, ρ) be a linear metric space and $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. A double sequence $x = (x_{mn})$ in X is said to be $\mathcal{I}_2$-convergent to $L \in X$, if for any $\varepsilon > 0$ we have $A(\varepsilon) = \{(m, n) \in \mathbb{N} \times \mathbb{N} : \rho(x_{mn}, L) \geq \varepsilon\} \in \mathcal{I}_2$ and is written $\mathcal{I}_2 - \lim_{m,n\to\infty} x_{mn} = L$.

If $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ is a strongly admissible ideal, then usual convergence implies $\mathcal{I}_2$-convergence.

Let $\mathcal{I}_2$ be an ideal of $\mathbb{N}\times\mathbb{N}$ and $\mathcal{I}$ be an ideal of $\mathbb{N}$, then a double sequence $x = (x_{mn})$ in $\mathbb{C}$, which is the set of complex numbers, is said to be regularly $(\mathcal{I}_2, \mathcal{I})$-convergent ($r(\mathcal{I}_2, \mathcal{I})$-convergent), if it is $\mathcal{I}_2$-convergent in Pringsheim's sense and for every $\varepsilon > 0$, the following statements hold: $\{m \in \mathbb{N} : |x_{mn} - L_n| \geq \varepsilon\} \in \mathcal{I}$ for some $L_n \in \mathbb{C}$, for each $n \in \mathbb{N}$ and $\{n \in \mathbb{N} : |x_{mn} - K_m| \geq \varepsilon\} \in \mathcal{I}$ for some $K_m \in \mathbb{C}$, for each $m \in \mathbb{N}$.

We say that an admissible ideal $\mathcal{I} \subset 2^{\mathbb{N}}$ satisfies the property (AP), if for every countable family of mutually disjoint sets $\{A_1, A_2, \ldots\}$ belonging to $\mathcal{I}$, there exists a countable family of sets $\{B_1, B_2, \ldots\}$ such that $A_j \Delta B_j$ is a finite set for $j \in \mathbb{N}$ and $B = \bigcup_{j=1}^{\infty} B_j \in \mathcal{I}$. (hence $B_j \in \mathcal{I}$ for each $j \in \mathbb{N}$).

We say that an admissible ideal $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ satisfies the property ($AP2$), if for every countable family of mutually disjoint sets $\{A_1, A_2, \ldots\}$ belonging to $\mathcal{I}_2$, there exists a countable family of sets $\{B_1, B_2, \ldots\}$ such that $A_j \Delta B_j \in \mathcal{I}_2^0$, i.e., $A_j \Delta B_j$ is included in the finite union of rows and columns in $\mathbb{N} \times \mathbb{N}$ for each $j \in \mathbb{N}$ and $B = \bigcup_{j=1}^{\infty} B_j \in \mathcal{I}_2$ (hence $B_j \in \mathcal{I}_2$ for each $j \in \mathbb{N}$).

Let X be a real vector space of dimension d, where $2 \leq d < \infty$. A 2-norm on X is a function $\|\cdot, \cdot\|$: $X \times X \to \mathbb{R}$ which satisfies (i) $\|x, y\| = 0$ if and only if x and y are linearly dependent; (ii) $\|x, y\| = \|y, x\|$; (iii) $\|\alpha x, y\| = |\alpha|\|x, y\|$, $\alpha \in \mathbb{R}$; (iv) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$. The pair $(X, \|\cdot, \cdot\|)$ is then called a 2-normed space. As an example of a 2-normed space we may take $X = \mathbb{R}^2$ being equipped with the 2-norm $\|x, y\|$:= the area of the parallelogram spanned by the vectors x and y, which may be given explicitly by the formula

$$\|x, y\| = |x_1 y_2 - x_2 y_1|, \quad x = (x_1, x_2), \quad y = (y_1, y_2).$$

The sequence $(x_n)_{n\in\mathbb{N}}$ in 2-normed space $(X, \|\cdot,\cdot\|)$ is said to be convergent to $L \in X$, if for each $\varepsilon > 0$ and nonzero $z \in X$, $\|x_n - L, z\| < \varepsilon$. In this case we write $\lim_{n\to\infty} \|x_n - L, z\| = 0$ or $\lim_{n\to\infty} x_n = L$.

The double sequence $(x_{mn})_{m,n\in\mathbb{N}}$ in 2-normed space $(X, \|\cdot,\cdot\|)$ is said to be convergent to $L \in X$ in Pringsheim's sense, if for each $\varepsilon > 0$ and nonzero $z \in X$, $\|x_{mn} - L, z\| < \varepsilon$. In this case we write $P - \lim_{m,n\to\infty} \|x_{mn} - L, z\| = 0$ or $P - \lim_{m,n\to\infty} x_{mn} = L$.

Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be a nontrivial ideal. The sequence (x_n) in 2-normed space $(X, \|\cdot,\cdot\|)$ is said to be $\mathcal{I}$-convergent to $L \in X$, if for each $\varepsilon > 0$ and nonzero $z \in X$, $A(\varepsilon) = \{n \in \mathbb{N} : \|x_n - L, z\| \geq \varepsilon\} \in \mathcal{I}$. In this case we write $\mathcal{I} - \lim_{n\to\infty} \|x_n - L, z\| = 0$ or $\mathcal{I} - \lim_{n\to\infty} x_n = L$.

Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be a nontrivial ideal. The sequence (x_n) in 2-normed space $(X, \|\cdot,\cdot\|)$ is said to be $\mathcal{I}^*$-convergent to $L \in X$, if there exists a set $M = \{m_1 < m_2 < \cdots < m_k < \cdots\} \subset \mathbb{N}$, $M \in F(\mathcal{I})$ such that $\lim_{k\to\infty} \|x_{m_k} - L, z\| = 0$, for each nonzero $z \in X$.

Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space and $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal. The sequence (x_n) is said to be $\mathcal{I}$-Cauchy sequence in X, if for each $\varepsilon > 0$ and nonzero $z \in X$ there exists a number $N = N(\varepsilon, z)$ such that $\{n \in \mathbb{N} : \|x_n - x_N, z\| \geq \varepsilon\} \in \mathcal{I}$.

Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space and $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal. The sequence (x_n) is said to be $\mathcal{I}^*$-Cauchy sequence in X, if there exists a set $M = \{m_1 < m_2 < \cdots < m_k < \cdots\} \subset \mathbb{N}$, $M \in F(\mathcal{I})$ such that $\lim_{k,p\to\infty} \|x_{m_k} - x_{m_p}, z\| = 0$, for each nonzero $z \in X$.

Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. A double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ in 2-normed space $(X, \|\cdot,\cdot\|)$ is said to be $\mathcal{I}_2$-convergent to $L \in X$, if for each $\varepsilon > 0$ and nonzero $z \in X$, $A(\varepsilon) = \{(m, n) \in \mathbb{N} \times \mathbb{N} : \|x_{mn} - L, z\| \geq \varepsilon\} \in \mathcal{I}_2$. In this case we write $\mathcal{I}_2 - \lim_{m,n\to\infty} x_{mn} = L$.

Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. A double sequence $x = (x_{mn})_{m,n\in\mathbb{N}}$ in 2-normed space $(X, \|\cdot,\cdot\|)$ is said to be $\mathcal{I}_2^*$-convergent to $L \in X$, if there exists a set $M \in F(\mathcal{I}_2)$ (i.e. $H = \mathbb{N} \times \mathbb{N} \backslash M \in \mathcal{I}_2$) such that $\lim_{m,n\to\infty} \|x_{mn} - L, z\| = 0$, for $(m, n) \in M$ and for each nonzero $z \in X$. In this case we write $\mathcal{I}_2^* - \lim_{m,n\to\infty} x_{mn} = L$.

Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space and $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. A double sequence $x = (x_{mn})$ in X is said to be $\mathcal{I}_2$-Cauchy if for each $\varepsilon > 0$ and nonzero z in X there exist $s = s(\varepsilon, z)$, $t = t(\varepsilon, z) \in \mathbb{N}$ such that

$$A(\varepsilon) := \{(m, n) \in \mathbb{N} \times \mathbb{N} : \|x_{mn} - x_{st}, z\| \geq \varepsilon\} \in \mathcal{I}_2.$$

Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space and $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. A double sequence $x = (x_{mn})$ in X is said to be $\mathcal{I}_2^*$-Cauchy sequence if there exists a set $M \in \mathcal{F}(\mathcal{I}_2)$ (i.e., $H = \mathbb{N} \times \mathbb{N} \backslash M \in \mathcal{I}_2$) such that for each $\varepsilon > 0$ and for all $(m, n), (s, t) \in M$,

$$\|x_{mn} - x_{st}, z\| < \varepsilon, \text{ for each nonzero z in X},$$

where $m, n, s, t > k_0 = k_0(\varepsilon) \in \mathbb{N}$. In this case we write

$$\lim_{m,n,s,t\to\infty} \|x_{mn} - x_{st}, z\| = 0.$$

Now, we begin with quoting the following lemmas due to Sarabadan et al. [24] and Dündar, Sever [5] which are needed throughout the paper.

Lemma 1.1. *[24, Theorem 4.3] Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal with property $(AP2)$ and $(X, \|\cdot,\cdot\|)$ be a finite dimensional 2-normed space, then for a double sequence $x = (x_{mn})$ of X, $\mathcal{I}_2 - \lim_{m,n\to\infty} x_{mn} = L$ implies $\mathcal{I}_2^* - \lim_{m,n\to\infty} x_{mn} = L$.*

Lemma 1.2. *[5, Theorem 3.2] Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space and $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. If $x = (x_{mn})$ in X is $\mathcal{I}_2$-convergent then $x = (x_{mn})$ is $\mathcal{I}_2$-Cauchy double sequence.*

Lemma 1.3. *[5, Theorem 3.4] Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space and $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. If $x = (x_{mn})$ in X is $\mathcal{I}_2^*$-Cauchy double sequence then $x = (x_{mn})$ is $\mathcal{I}_2$-Cauchy double sequence.*

2. Main Results

The proof of the following lemma is similar to the proof of [2, Theorem 1], so we omit it.

Lemma 2.1. *Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space and $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. Then for $x = (x_{mn})$ be a double sequence of X, $\mathcal{I}_2^* - \lim_{m,n\to\infty} x_{mn} = L$ implies $\mathcal{I}_2 - \lim_{m,n\to\infty} x_{mn} = L$.*

Lemma 2.2. *Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space and $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal. Then for $x = (x_{mn})$ be a double sequence of X, $L \in X$ and for each nonzero $z \in X$,*

$$P - \lim_{m,n\to\infty} \|x_{mn} - L, z\| = 0 \ \text{ implies } \ \mathcal{I}_2 - \lim_{m,n\to\infty} \|x_{mn} - L, z\| = 0.$$

Proof. Let

$$P - \lim_{m,n\to\infty} \|x_{mn} - L, z\| = 0.$$

For each $\varepsilon > 0$ and nonzero $z \in X$ there exists $k_0 = k_0(\varepsilon) \in \mathbb{N}$ such that $\|x_{mn} - L, z\| < \varepsilon$ for all $m, n \geq k_0$. Then,

$$\begin{aligned} A(\varepsilon) &= \{(m,n) \in \mathbb{N}\times\mathbb{N} : \|x_{mn} - L, z\| \geq \varepsilon\} \\ &\subset \Big(\mathbb{N}\times\{1,2,\ldots,(k_0-1)\} \cup \{1,2,\ldots,(k_0-1)\}\times\mathbb{N}\Big). \end{aligned}$$

Since $\mathcal{I}_2$ is a strongly admissible ideal we have $\Big(\mathbb{N}\times\{1,2,\ldots,(k_0-1)\} \cup \{1,2,\ldots,(k_0-1)\}\times\mathbb{N}\Big) \in \mathcal{I}_2$ and so $A(\varepsilon) \in \mathcal{I}_2$. Hence, this completes the proof. □

Now, we study certain properties of regularly convergence, regularly $(\mathcal{I}_2, \mathcal{I})$-convergence and regularly $(\mathcal{I}_2, \mathcal{I})$-Cauchy double sequences in 2-normed spaces.

Definition 2.3. *Let $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space. A double sequence (x_{mn}) in X is said to be regularly convergent, if it is convergent in Pringsheim's sense and the limits*

$$\lim_{m\to\infty} x_{mn},\ (n \in \mathbb{N}) \ \text{ and } \ \lim_{n\to\infty} x_{mn},\ (m \in \mathbb{N}),$$

exist for each fixed $n \in \mathbb{N}$ and $m \in \mathbb{N}$, respectively. Note that if (x_{mn}) is regularly convergent to L in X, then the limits

$$\lim_{n\to\infty}\lim_{m\to\infty} x_{mn} \ \text{ and } \ \lim_{m\to\infty}\lim_{n\to\infty} x_{mn}$$

exist and are equal to L. In this case we write

$$r - \lim_{m,n\to\infty} x_{mn} = L \ \text{ or } \ x_{mn} \xrightarrow{r} L.$$

Definition 2.4. *Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal, $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal and $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space. A double sequence (x_{mn}) in X is said to be regularly $(\mathcal{I}_2, \mathcal{I})$-convergent ($r(\mathcal{I}_2, \mathcal{I})$-convergent), if it is $\mathcal{I}_2$-convergent in Pringsheim's sense and for each $\varepsilon > 0$ and nonzero $z \in X$, the following statements hold:*

$$\{m \in \mathbb{N} : \|x_{mn} - L_n, z\| \geq \varepsilon\} \in \mathcal{I} \tag{1}$$

for some $L_n \in X$, for each $n \in \mathbb{N}$ and

$$\{n \in \mathbb{N} : \|x_{mn} - K_m, z\| \geq \varepsilon\} \in \mathcal{I} \tag{2}$$

for some $K_m \in X$, for each $m \in \mathbb{N}$.

If (x_{mn}) is regularly $(\mathcal{I}_2, \mathcal{I})$-convergent ($r(\mathcal{I}_2, \mathcal{I})$-convergent) to $L \in X$, then the limits $\mathcal{I} - \lim_{n\to\infty}\lim_{m\to\infty} x_{mn}$ and $\mathcal{I} - \lim_{m\to\infty}\lim_{n\to\infty} x_{mn}$ exist and are equal to L.

Theorem 2.5. *Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal, $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal and $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space. If a double sequence (x_{mn}) in X is regularly convergent, then (x_{mn}) is $r(\mathcal{I}_2, \mathcal{I})$-convergent.*

Proof. Let (x_{mn}) be regularly convergent. Then (x_{mn}) is convergent in Pringsheim's sense and the limits $\lim_{m\to\infty} x_{mn}$ $(n \in \mathbb{N})$ and $\lim_{n\to\infty} x_{mn}$ $(m \in \mathbb{N})$ exist. By Lemma 2.2, (x_{mn}) is $\mathcal{I}_2$-convergent. Also, for each $\varepsilon > 0$ and nonzero $z \in X$, there exist $m = m_0(\varepsilon)$ and $n = n_0(\varepsilon)$ such that

$$\|x_{mn} - L_n, z\| < \varepsilon$$

for some L_n and each fixed $n \in \mathbb{N}$ for every $m \geq m_0$ and

$$\|x_{mn} - K_m, z\| < \varepsilon$$

for some K_m and each fixed $m \in \mathbb{N}$ for every $n \geq n_0$. Then, since $\mathcal{I}$ is an admissible ideal so for each $\varepsilon > 0$ and nonzero $z \in X$, we have

$$\{m \in \mathbb{N} : \|x_{mn} - L_n, z\| \geq \varepsilon\} \subset \{1, 2, \ldots, m_0 - 1\} \in \mathcal{I},$$

$$\{n \in \mathbb{N} : \|x_{mn} - K_m, z\| \geq \varepsilon\} \subset \{1, 2, \ldots, n_0 - 1\} \in \mathcal{I}.$$

Hence, (x_{mn}) is $r(\mathcal{I}_2, \mathcal{I})$-convergent in X. □

Definition 2.6. *Let $\mathcal{I}_2$ be a strongly admissible ideal of $\mathbb{N}\times\mathbb{N}$, $\mathcal{I}$ be an admissible ideal of $\mathbb{N}$ and $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space. A double sequence (x_{mn}) in X is said to be $r(\mathcal{I}_2^*, \mathcal{I}^*)$-convergent, if there exist the sets $M \in \mathcal{F}(\mathcal{I}_2)$ (i.e., $\mathbb{N}\times\mathbb{N} \setminus M \in \mathcal{I}_2$), $M_1 \in \mathcal{F}(\mathcal{I})$ and $M_2 \in \mathcal{F}(\mathcal{I})$ (i.e., $\mathbb{N} \setminus M_1 \in \mathcal{I}$ and $\mathbb{N} \setminus M_2 \in \mathcal{I}$) such that the limits*

$$\lim_{\substack{m,n\to\infty \\ (m,n)\in M}} x_{mn}, \quad \lim_{\substack{m\to\infty \\ m\in M_1}} x_{mn} \quad \text{and} \quad \lim_{\substack{n\to\infty \\ n\in M_2}} x_{mn}$$

exist for each fixed $n \in \mathbb{N}$ and $m \in \mathbb{N}$, respectively.

Theorem 2.7. *Let $\mathcal{I}_2$ be a strongly admissible ideal of $\mathbb{N}\times\mathbb{N}$, $\mathcal{I}$ be an admissible ideal of $\mathbb{N}$ and $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space. If a double sequence (x_{mn}) in X is $r(\mathcal{I}_2^*, \mathcal{I}^*)$-convergent, then it is $r(\mathcal{I}_2, \mathcal{I})$-convergent.*

Proof. Let (x_{mn}) in X be $r(\mathcal{I}_2^*, \mathcal{I}^*)$-convergent. Then, it is $\mathcal{I}_2^*$-convergent and so, by Lemma 2.1, it is $\mathcal{I}_2$-convergent. Also, there exist the sets $M_1, M_2 \in \mathcal{F}(\mathcal{I})$ such that

$$(\forall z \in X)\ (\forall \varepsilon > 0)\ (\exists m_0 \in \mathbb{N})\ (\forall m \geq m_0)\ (m \in M_1)\ \|x_{mn} - L_n, z\| < \varepsilon,\ \ (n \in \mathbb{N})$$

for some $L_n \in X$ and

$$(\forall z \in X)\ (\forall \varepsilon > 0)\ (\exists n_0 \in \mathbb{N})\ (\forall n \geq n_0)\ (n \in M_2)\ \|x_{mn} - K_m, z\| < \varepsilon,\ \ (m \in \mathbb{N})$$

for some $K_m \in X$. Hence, for each $\varepsilon > 0$ and nonzero $z \in X$, we have

$$\begin{aligned} A(\varepsilon) &= \{m \in \mathbb{N} : \|x_{mn} - L_n, z\| \geq \varepsilon\} \subset H_1 \cup \{1, 2, \ldots, m_0 - 1\},\ \ (n \in \mathbb{N}), \\ B(\varepsilon) &= \{n \in \mathbb{N} : \|x_{mn} - K_m, z\| \geq \varepsilon\} \subset H_2 \cup \{1, 2, \ldots, n_0 - 1\},\ \ (m \in \mathbb{N}), \end{aligned}$$

for $H_1, H_2 \in \mathcal{I}$. Since $\mathcal{I}$ is an admissible ideal we get

$$H_1 \cup \{1, 2, \ldots, (m_0 - 1)\} \in \mathcal{I},\ \ H_2 \cup \{1, 2, \ldots, n_0 - 1\} \in \mathcal{I}$$

and therefore $A(\varepsilon), B(\varepsilon) \in \mathcal{I}$. This shows that the double sequence (x_{mn}) is $r(\mathcal{I}_2, \mathcal{I})$-convergent in X. □

Theorem 2.8. *Let $\mathcal{I}_2 \subset 2^{\mathbb{N}\times\mathbb{N}}$ be a strongly admissible ideal with property (AP2), $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal with property (AP) and $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space. If a double sequence (x_{mn}) is $r(\mathcal{I}_2, \mathcal{I})$-convergent, then (x_{mn}) is $r(\mathcal{I}_2^*, \mathcal{I}^*)$-convergent in X.*

Proof. Let a double sequence (x_{mn}) in X be $r(\mathcal{I}_2, \mathcal{I})$-convergent. Then (x_{mn}) is $\mathcal{I}_2$-convergent and so (x_{mn}) is $\mathcal{I}_2^*$-convergent, by Lemma 1.1. Also, for each $\varepsilon > 0$ and nonzero $z \in X$ we have

$$A(\varepsilon) = \{m \in \mathbb{N} : \|x_{mn} - L_n, z\| \geq \varepsilon\} \in \mathcal{I}$$

for some $L_n \in X$, for each $n \in \mathbb{N}$ and

$$C(\varepsilon) = \{n \in \mathbb{N} : \|x_{mn} - K_m, z\| \geq \varepsilon\} \in \mathcal{I}$$

for some $K_m \in X$, for each $m \in \mathbb{N}$.

Now put for each nonzero $z \in X$

$$\begin{aligned} A_1 &= \{m \in \mathbb{N} : \|x_{mn} - L_n, z\| \geq 1\}, \\ A_k &= \left\{m \in \mathbb{N} : \frac{1}{k} \leq \|x_{mn} - L_n, z\| < \frac{1}{k-1}\right\} \end{aligned}$$

for $k \geq 2$, for some $L_n \in X$ and for each $n \in \mathbb{N}$. It is clear that $A_i \cap A_j = \emptyset$ for $i \neq j$ and $A_i \in \mathcal{I}$ for each $i \in \mathbb{N}$. By the property (AP) there is a countable family of sets $\{B_1, B_2, \ldots\}$ in $\mathcal{I}$ such that $A_j \vartriangle B_j$ is a finite set for each $j \in \mathbb{N}$ and $B = \bigcup_{j=1}^{\infty} B_j \in \mathcal{I}$.

We prove that

$$\lim_{\substack{m\to\infty \\ m\in M}} \|x_{mn} - L_n, z\| = 0, \text{ for some } L_n \text{ and for each } n \in \mathbb{N}$$

for each nonzero $z \in X$ and for $M = \mathbb{N}\backslash B \in \mathcal{F}(\mathcal{I})$. Let $\delta > 0$ be given. Choose $k \in \mathbb{N}$ such that $1/k < \delta$. Then, for each nonzero $z \in X$ we have

$$\{m \in \mathbb{N} : \|x_{mn} - L_n, z\| \geq \delta\} \subset \bigcup_{j=1}^{k} A_j \text{ for some } L_n \text{ and for each } n \in \mathbb{N}.$$

Since $A_j \vartriangle B_j$ is a finite set for $j \in \{1, 2, \ldots, k\}$, there exists $m_0 \in \mathbb{N}$ such that

$$\Big(\bigcup_{j=1}^{k} B_j\Big) \cap \{m : m \geq m_0\} = \Big(\bigcup_{j=1}^{k} A_j\Big) \cap \{m : m \geq m_0\}.$$

If $m \geq m_0$ and $m \notin B$ then

$$m \notin \bigcup_{j=1}^{k} B_j \text{ and so } m \notin \bigcup_{j=1}^{k} A_j.$$

Thus, for each nonzero $z \in X$ we have $\|x_{mn} - L_n, z\| < \frac{1}{k} < \delta$ for some L_n and for each $n \in \mathbb{N}$. This implies that

$$\lim_{\substack{m\to\infty \\ m\in M}} \|x_{mn} - L_n, z\| = 0.$$

Hence, for each nonzero $z \in X$ we have

$$\mathcal{I}^* - \lim_{m\to\infty} \|x_{mn} - L_n, z\| = 0$$

for some L_n and for each $n \in \mathbb{N}$.

Similarly, for the set $C(\varepsilon) = \{n \in \mathbb{N} : \|x_{mn} - K_m, z\| \geq \varepsilon\} \in \mathcal{I}$, for each nonzero $z \in X$ we have

$$\mathcal{I}^* - \lim_{n\to\infty} \|x_{mn} - K_m, z\| = 0$$

for K_m and for each $m \in \mathbb{N}$. Hence, a double sequence (x_{mn}) is $r(\mathcal{I}_2^*, \mathcal{I}^*)$-convergent. □

Now, we give the definitions of $r(\mathcal{I}_2, \mathcal{I})$-Cauchy sequence and $r(\mathcal{I}_2^*, \mathcal{I}^*)$-Cauchy sequence.

Definition 2.9. *Let $\mathcal{I}_2$ be a strongly admissible ideal of $\mathbb{N}\times\mathbb{N}$, $\mathcal{I}$ be an admissible ideal of $\mathbb{N}$ and $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space. A double sequence (x_{mn}) in X is said to be regularly $(\mathcal{I}_2,\mathcal{I})$-Cauchy ($r(\mathcal{I}_2,\mathcal{I})$-Cauchy), if it is $\mathcal{I}_2$-Cauchy in Pringsheim's sense and for each $\varepsilon > 0$ and nonzero $z \in X$ there exist $k_n = k_n(\varepsilon, z) \in \mathbb{N}$ and $l_m = l_m(\varepsilon, z) \in \mathbb{N}$ such that the following statements hold:*

$$\begin{aligned} A_1(\varepsilon) &= \{m \in \mathbb{N} : \|x_{mn} - x_{k_n n}, z\| \geq \varepsilon\} \in \mathcal{I}, \ (n \in \mathbb{N}), \\ A_2(\varepsilon) &= \{n \in \mathbb{N} : \|x_{mn} - x_{m l_m}, z\| \geq \varepsilon\} \in \mathcal{I}, \ (m \in \mathbb{N}). \end{aligned}$$

Definition 2.10. *Let $\mathcal{I}_2$ be a strongly admissible ideal of $\mathbb{N}\times\mathbb{N}$, $\mathcal{I}$ be an admissible ideal of $\mathbb{N}$ and $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space. A double sequence (x_{mn}) is said to be regularly $(\mathcal{I}_2^*,\mathcal{I}^*)$-Cauchy ($r(\mathcal{I}_2^*,\mathcal{I}^*)$-Cauchy), if there exist the sets $M \in \mathcal{F}(\mathcal{I}_2)$, $M_1 \in \mathcal{F}(\mathcal{I})$ and $M_2 \in \mathcal{F}(\mathcal{I})$ (i.e., $\mathbb{N}\times\mathbb{N}\setminus M \in \mathcal{I}_2$, $\mathbb{N}\setminus M_1 \in \mathcal{I}$ and $\mathbb{N}\setminus M_2 \in \mathcal{I}$), for each $\varepsilon > 0$ and nonzero $z \in X$ there exist $N = N(\varepsilon)$, $s = s(\varepsilon)$, $t = t(\varepsilon)$, $(s,t) \in M$, $k_n = k_n(\varepsilon)$, $l_m = l_m(\varepsilon) \in \mathbb{N}$ such that*

$$\begin{aligned} &\|x_{mn} - x_{st}, z\| < \varepsilon, \ \text{for } (m,n),(s,t) \in M, \\ &\|x_{mn} - x_{k_n n}, z\| < \varepsilon, \ \text{for each } m \in M_1 \text{ and for each } n \in \mathbb{N}, \\ &\|x_{mn} - x_{m l_m}, z\| < \varepsilon, \ \text{for each } n \in M_2 \text{ and for each } m \in \mathbb{N}, \end{aligned}$$

whenever $m, n, s, t, k_n, l_m \geq N$.

Theorem 2.11. *Let $\mathcal{I}_2$ be a strongly admissible ideal of $\mathbb{N}\times\mathbb{N}$ and $\mathcal{I}$ be an admissible ideal of $\mathbb{N}$ and $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space. If a double sequence (x_{mn}) in X is $r(\mathcal{I}_2^*,\mathcal{I}^*)$-Cauchy, then it is $r(\mathcal{I}_2,\mathcal{I})$-Cauchy.*

Proof. Since a double sequence (x_{mn}) in X is $r(\mathcal{I}_2^*,\mathcal{I}^*)$-Cauchy, it is $\mathcal{I}_2^*$-Cauchy. We know that $\mathcal{I}_2^*$-Cauchy implies $\mathcal{I}_2$-Cauchy by Lemma 1.3. Also, since the double sequence (x_{mn}) is $r(\mathcal{I}_2^*,\mathcal{I}^*)$-Cauchy so there exist the sets $M_1, M_2 \in \mathcal{F}(\mathcal{I})$ and for each $\varepsilon > 0$ and nonzero $z \in X$ there exist $k_n = k_n(\varepsilon) \in \mathbb{N}$ and $l_m = l_m(\varepsilon) \in \mathbb{N}$ such that

$$\begin{aligned} \|x_{mn} - x_{k_n n}, z\| &< \varepsilon, \ \text{for each } m \in M_1 \text{ and for each } n \in \mathbb{N}, \\ \|x_{mn} - x_{m l_m}, z\| &< \varepsilon, \ \text{for each } n \in M_2 \text{ and for each } m \in \mathbb{N}, \end{aligned}$$

for $N = N(\varepsilon) \in \mathbb{N}$ and $m, n, k_n, l_m \geq N$. Therefore, for $H_1 = \mathbb{N}\setminus M_1 \in \mathcal{I}, H_2 = \mathbb{N}\setminus M_2 \in \mathcal{I}$ we have

$$A_1(\varepsilon) = \{m \in \mathbb{N} : \|x_{mn} - x_{k_n n}, z\| \geq \varepsilon\} \subset H_1 \cup \{1, 2, \ldots, N-1\}, \ (n \in \mathbb{N})$$

for $m \in M_1$ and

$$A_2(\varepsilon) = \{n \in \mathbb{N} : \|x_{mn} - x_{m l_m}, z\| \geq \varepsilon\} \subset H_2 \cup \{1, 2, \ldots, N-1\}, \ (m \in \mathbb{N})$$

for $n \in M_2$. Since $\mathcal{I}$ is an admissible ideal,

$$H_1 \cup \{1, 2, \ldots, N-1\} \in \mathcal{I} \text{ and } H_2 \cup \{1, 2, \ldots, N-1\} \in \mathcal{I}.$$

Hence, we have $A_1(\varepsilon), A_2(\varepsilon) \in \mathcal{I}$ and (x_{mn}) is $r(\mathcal{I}_2,\mathcal{I})$-Cauchy double sequence. □

Theorem 2.12. *Let $\mathcal{I}_2$ be a strongly admissible ideal of $\mathbb{N}\times\mathbb{N}$ and $\mathcal{I}$ be an admissible ideal of $\mathbb{N}$ and $(X, \|\cdot,\cdot\|)$ be a linear 2-normed space. If a double sequence (x_{mn}) in X is $r(\mathcal{I}_2,\mathcal{I})$-convergent, then (x_{mn}) is $r(\mathcal{I}_2,\mathcal{I})$-Cauchy double sequence.*

Proof. Let (x_{mn}) be a $r(\mathcal{I}_2,\mathcal{I})$-convergent double sequence in X. Then (x_{mn}) is $\mathcal{I}_2$-convergent and by Lemma 1.2, it is $\mathcal{I}_2$-Cauchy double sequence. Also for each $\varepsilon > 0$ and nonzero $z \in X$, we have

$$A_1\Big(\frac{\varepsilon}{2}\Big) = \Big\{m \in \mathbb{N} : \|x_{mn} - L_n, z\| \geq \frac{\varepsilon}{2}\Big\} \in \mathcal{I}$$

for some L_n, for each $n \in \mathbb{N}$ and

$$A_2\Big(\frac{\varepsilon}{2}\Big) = \Big\{n \in \mathbb{N} : \|x_{mn} - K_m, z\| \geq \frac{\varepsilon}{2}\Big\} \in \mathcal{I}$$

for some K_m, for each $m \in \mathbb{N}$. Since $\mathcal{I}$ is an admissible ideal, the sets

$$A_1^c\Big(\frac{\varepsilon}{2}\Big) = \Big\{m \in \mathbb{N} : \|x_{mn} - L_n, z\| < \frac{\varepsilon}{2}\Big\},\ (n \in \mathbb{N})$$

for some L_n and

$$A_2^c\Big(\frac{\varepsilon}{2}\Big) = \Big\{n \in \mathbb{N} : \|x_{mn} - K_m, z\| < \frac{\varepsilon}{2}\Big\},\ (m \in \mathbb{N})$$

for some K_m, are nonempty and belong to $\mathcal{F}(\mathcal{I})$. For $k_n \in A_1^c(\frac{\varepsilon}{2})$, ($n \in \mathbb{N}$ and $k_n > 0$) we have

$$\|x_{k_n n} - L_n, z\| < \frac{\varepsilon}{2}$$

for some L_n. Now, for each $\varepsilon > 0$ and nonzero $z \in X$ we define the set

$$B_1(\varepsilon) = \{m \in \mathbb{N} : \|x_{mn} - x_{k_n n}, z\| \geq \varepsilon\},\ \ (n \in \mathbb{N}),$$

where $k_n = k_n(\varepsilon) \in \mathbb{N}$. Let $m \in B_1(\varepsilon)$. Then for $k_n \in A_1^c(\frac{\varepsilon}{2})$, ($n \in \mathbb{N}$ and $k_n > 0$) we have

$$\begin{aligned} \varepsilon \leq \|x_{mn} - x_{k_n n}, z\| &\leq \|x_{mn} - L_n, z\| + \|x_{k_n n} - L_n, z\| \\ &< \|x_{mn} - L_n, z\| + \frac{\varepsilon}{2} \end{aligned}$$

for some L_n. This shows that

$$\frac{\varepsilon}{2} < \|x_{mn} - L_n, z\| \text{ and so } m \in A_1(\frac{\varepsilon}{2}).$$

Hence, we have $B_1(\varepsilon) \subset A_1(\frac{\varepsilon}{2})$.

Similarly, for each $\varepsilon > 0$, nonzero $z \in X$ and for $l_m \in A_2^c(\frac{\varepsilon}{2})$ ($m \in \mathbb{N}$ and $l_m > 0$) we have

$$\|x_{ml_m} - K_m, z\| < \frac{\varepsilon}{2},\ (m \in \mathbb{N})$$

for some K_m. Therefore, it can be seen that

$$B_2(\varepsilon) = \{m \in \mathbb{N} : \|x_{ml_m} - K_m, z\| \geq \varepsilon\} \subset A_2(\frac{\varepsilon}{2}).$$

Hence, we have $B_1(\varepsilon), B_2(\varepsilon) \in \mathcal{I}$. This shows that (x_{mn}) is $r(\mathcal{I}_2, \mathcal{I})$-Cauchy double sequence. □

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