ON STATISTICAL CONVERGENCE OF SEQUENCES OF FUNCTIONS IN 2-NORMED SPACES

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Abstract. Statistical convergence and statistical Cauchy sequence in 2-normed space were studied by Gürdal and Pehlivan [M. Gürdal, S. Pehlivan, *Statistical convergence in 2-normed spaces*, Southeast Asian Bulletin of Mathematics, (33) (2009), 257–264]. In this paper, we get analogous results of statistical convergence and statistical Cauchy sequence of functions and investigate some properties and relationships between them in 2-normed spaces.

1. Introduction

Throughout the paper, $\mathbb{N}$ denotes the set of all positive integers, $\mathbb{R}$ the set of all real numbers. The concept of convergence of a sequence of real numbers has been extended to statistical convergence independently by Fast [7] and Schoenberg [24]. Gökhan et al. [12] introduced the concepts of pointwise statistical convergence and statistical Cauchy sequence of real-valued functions. Balcerzak et al. [2] studied statistical convergence and ideal convergence for sequence of functions. Baláz et al. [1] investigated $\mathcal{I}$-convergence and $\mathcal{I}$-continuity of real functions. Gezer and Karakuş [11] investigated $\mathcal{I}$-pointwise and uniform convergence and $\mathcal{I}^*$-pointwise and uniform convergence of function sequences and then they examined the relation between them. Gökhan et al. [13] introduced the notion of pointwise and uniform statistical convergence of double sequences of real-valued functions. Dündar and Altay [4, 5] studied the concepts of pointwise and uniformly $\mathcal{I}$-convergence and $\mathcal{I}^*$-convergence of double sequences of functions and investigated some properties about them. Furthermore, Dündar [6] investigated some results of $\mathcal{I}_2$-convergence of double sequences of functions.

The concept of 2-normed spaces was initially introduced by Gähler [9, 10] in the 1960's. Gürdal and Pehlivan [16] studied statistical convergence, statistical Cauchy sequence and investigated some properties of statistical convergence in 2-normed spaces. Sharma and Kumar [22] introduced statistical convergence, statistical Cauchy sequence, statistical limit points and statistical cluster points in probabilistic 2-normed space. Savaş and Gürdal [23] concerned with $\mathcal{I}$-convergence of sequences of functions in random 2-normed spaces and introduce the concepts of ideal uniform convergence and ideal pointwise convergence in the topology induced by random 2-normed spaces. Sarabadan and Talebi [21] presented various kinds of statistical convergence and $\mathcal{I}$-convergence for sequences of functions with values in 2-normed spaces and also defined the notion of $\mathcal{I}$-equistatistically convergence and study $\mathcal{I}$-equistatistically convergence of sequences of functions. Şahiner et al. [25] and Gürdal [18] studied $\mathcal{I}$-convergence in 2-normed spaces. Gürdal and Açık [17] investigated $\mathcal{I}$-Cauchy and $\mathcal{I}^*$-Cauchy sequences in 2-normed spaces. Futhermore, a lot of development have been made in this area (see [3, 14, 15, 19, 20]).

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2. Definitions and Notations

Now, we recall the concept of density, statistical convergence, 2-normed space and some fundamental definitions and notations (See [2, 8, 10, 11, 12, 14, 15, 16, 21, 22]).

If $K \subseteq \mathbb{N}$, then K_n denotes the set $\{k \in K : k \leq n\}$ and $|K_n|$ denotes the cardinality of K_n. The natural density of K is given by $\delta(K) = \lim_{n\to\infty} \frac{1}{n}|K_n|$, if it exists.

Clearly, finite subsets have natural density zero and $\delta(K^c) = 1 - \delta(K)$ where $K^c = \mathbb{N}\backslash K$, i.e., the complement of K. If $K_1 \subseteq K_2$ and K_1 and K_2 have natural densities then $\delta(K_1) \leq \delta(K_2)$. Moreover, if $\delta(K_1) = \delta(K_2) = 1$, then $\delta(K_1 \cap K_2) = 1$.

The number sequence $x = (x_k)$ is statistically convergent to L provided that for every $\varepsilon > 0$ the set

$$K = K(\varepsilon) := \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$$

has natural density zero; in this case, we write $st - \lim x = L$.

We note following theorem which is useful in establishing our results.

Theorem 2.1. [8] *The following statements are equivalent:*

(i) x is statistically convergent sequence;

(ii) x is statistically Cauchy sequence;

(iii) x is sequence for which there is a convergent sequence y such that $x_n = y_n$, for a.a. n.

Let X be a real vector space of dimension d, where $2 \leq d < \infty$. A 2-norm on X is a function $\|\cdot,\cdot\| : X \times X \to \mathbb{R}$ which satisfies the following statements:

(i) $\|x, y\| = 0$ if and only if x and y are linearly dependent.

(ii) $\|x, y\| = \|y, x\|$.

(iii) $\|\alpha x, y\| = |\alpha|\|x, y\|$, $\alpha \in \mathbb{R}$.

(iv) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$.

The pair $(X, \|\cdot,\cdot\|)$ is then called a 2-normed space. As an example of a 2-normed space we may take $X = \mathbb{R}^2$ being equipped with the 2-norm $\|x, y\| :=$ the area of the parallelogram based on the vectors x and y which may be given explicitly by the formula

$$\|x, y\| = |x_1y_2 - x_2y_1|; \quad x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2.$$

In this study, we suppose X to be a 2-normed space having dimension d; where $2 \leq d < \infty$.

Let $(X, \|.,.\|)$ be a finite dimensional 2-normed space and $u = \{u_1, \cdots, u_d\}$ be a basis of X. We can define the norm $\|.\|_\infty$ on X by

$$\|x\|_\infty = \max\{\|x, u_i\| : i = 1, ..., d\}.$$

Associated to the derived norm $\|.\|_\infty$, we can define the (closed) balls $B_u(x, \varepsilon)$ centered at x having radius ε by

$$B_u(x, \varepsilon) = \{y : \|x - y\|_\infty \leq \varepsilon\},$$

where $\|x - y\|_\infty = \max\{\|x - y, u_j\|, j = 1, ..., d\}$.

Let X be a 2-normed space. A sequence (x_n) in X is said to be convergent to $L \in X$, if for every $z \in X$,

$$\lim_{n\to\infty} \|x_n - L, z\| = 0.$$

In this case, we write $\lim_{n\to\infty} x_n = L$ and call L the limit of (x_n).

Let $\{x_n\}$ be a sequence in 2-normed space $(X, \|.,.\|)$. The sequence (x_n) is said to be statistically convergent to L, if for every $\varepsilon > 0$, the set

$$\{n \in \mathbb{N} : \|x_n - L, z\| \geq \varepsilon\}$$

has natural density zero for each nonzero z in X, in other words (x_n) statistically converges to L in 2-normed space $(X, \|.,.\|)$ if

$$\lim_{n\to\infty} \frac{1}{n}\big|\{n : \|x_n - L, z\| \geq \varepsilon\}\big| = 0,$$

for each nonzero z in X. It means that for each $z \in X$,

$$\|x_n - L, z\| < \varepsilon\,,\ \ a.a.\ n.$$

In this case we write $st - \lim_{n\to\infty} \|x_n, z\| = \|L, z\|$.

A sequence (x_n) in 2-normed space $(X, \|.,.\|)$ is said to be statistically Cauchy sequence in X, if for every $\varepsilon > 0$ and every nonzero $z \in X$ there exists a number $N = N(\varepsilon, z)$ such that

$$\delta\big(\{n \in \mathbb{N} : \|x_n - x_N, z\| \geq \varepsilon\}\big) = 0,$$

i.e., for each nonzero $z \in X$,

$$\|x_n - x_N, z\| < \varepsilon,\ \text{a.a. n.}$$

Let X and Y be two 2-normed spaces and assume that functions $f_n : X \to Y$ and $f : X \to Y$ are given. The sequence of functions $\{f_n\}_{n\in\mathbb{N}}$ is said to be convergent to f if $f_n(x) \overset{\|.,.\|_Y}{\longrightarrow} f(x)$ for each $x \in X$. We write $f_n \overset{\|.,.\|_Y}{\longrightarrow} f$. This can be expressed by the formula

$$(\forall y \in Y)(\forall x \in X)(\forall \varepsilon > 0)(\exists n_0 \in \mathbb{N})(\forall n \geq n_0)\|f_n(x) - f(x, y)\| < \varepsilon.$$

3. Main Results

In this paper, we study concepts of convergence, statistical convergence and statistical Cauchy sequence of functions and investigate some properties and relationships between them in 2-normed spaces.

Throughout the paper, we let X and Y be two 2-normed spaces, $\{f_n\}_{n\in\mathbb{N}}$ and $\{g_n\}_{n\in\mathbb{N}}$ be two sequences of functions and f, g be two functions from X to Y.

Definition 3.1. *The sequence $\{f_n\}_{n\in\mathbb{N}}$ is said to be (pointwise) statistical convergent to f, if for every $\varepsilon > 0$,*

$$\lim_{n\to\infty} \frac{1}{n}\big|\{n \in \mathbb{N} : \|f_n(x) - f(x), z\| \geq \varepsilon\}\big| = 0,$$

for each $x \in X$ and each nonzero $z \in Y$. It means that for each $x \in X$ and each nonzero $z \in Y$,

$$\|f_n(x) - f(x), z\| < \varepsilon, \quad a.a.\ n.$$

In this case, we write

$$st - \lim_{n\to\infty} \|f_n(x) - z\| = \|f(x), z\| \quad or \quad f_n \overset{\|.,.\|_Y}{\longrightarrow}_{st} f.$$

Remark 3.1. *$\{f_n\}_{n\in\mathbb{N}}$ is any sequence of functions and f is any function from X to Y, then set*

$$\{n \in \mathbb{N} : \|f_n(x) - f(x), z\| \geq \varepsilon,\ \ for\ \ each\ \ x \in X\ \ and\ \ each\ \ z \in Y\} = \emptyset,$$

since if $z = \overrightarrow{0}$ (0 vektor), $\|f_n(x) - f(x), z\| = 0 \ngeq \varepsilon$ so the above set is empty.

Theorem 3.1. *If for each $x \in X$ and each nonzero $z \in Y$,*

$$st - \lim_{n\to\infty} \|f_n(x), z\| = \|f(x), z\| \quad and \quad st - \lim_{n\to\infty} \|f_n(x), z\| = \|g(x), z\|,$$

then $\|f_n(x), z\| = \|g_n(x), z\|$ (i.e., $f = g$), for each $x \in X$ and each nonzero $z \in Y$.

Proof 3.1. *Assume $f \neq g$. Then $f - g \neq \overrightarrow{0}$, so there exists a $z \in Y$ such that f, g and z are linearly independent (such a z exists since $d \geq 2$). Therefore, for each $x \in X$ and each nonzero $z \in Y$,*

$$\|f(x) - g(x), z\| = 2\varepsilon, \quad with \quad \varepsilon > 0.$$

Now, for each $x \in X$ and each nonzero $z \in Y$, we get

$$\begin{aligned} 2\varepsilon = \|f(x) - g(x), z\| &= \|(f(x) - f_n(x)) + (f_n(x) - g(x)), z\| \\ &\leq \|f_n(x) - g(x), z\| + \|f_n(x) - f(x), z\| \end{aligned}$$

and so

$$\{n : \|f_n(x) - g(x), z\| < \varepsilon\} \subseteq \{n : \|f_n(x) - f(x), z\| \geq \varepsilon\}.$$

But, for each $x \in X$ and each nonzero $z \in Y$, $\delta(\{n : \|f_n(x) - g(x), z\| < \varepsilon\}) = 0$, then contradicting the fact that $f_n \overset{\|.,.\|_Y}{\longrightarrow}_{st} g$.

Theorem 3.2. *If $\{g_n\}_{(n\in\mathbb{N})}$ is a convergent sequence of functions such that $f_n = g_n$ a.a. n, then $\{f_n\}_{(n\in\mathbb{N})}$ is statistically convergent.*

Proof 3.2. *Suppose that for each $x \in X$ and each nonzero $z \in Y$,*

$$\delta(\{n \in \mathbb{N} : f_n(x) \neq g_n(x)\}) = 0 \quad and \quad \lim_{n\to\infty} \|g_n(x), z\| = \|f(x), z\|,$$

then for every $\varepsilon > 0$,

$$\{n \in \mathbb{N} : \|f_n(x) - f(x), z\| \geq \varepsilon\} \subseteq \{n \in \mathbb{N} : \|g_n(x) - f(x), z\| \geq \varepsilon\} \cup \{n \in \mathbb{N} : f_n(x) \neq g_n(x)\}.$$

Therefore,

$$\begin{aligned} \delta(\{n \in \mathbb{N} : \|f_n(x) - f(x), z\| \geq \varepsilon\}) &\leq \delta(\{n \in \mathbb{N} : \|g_n(x) - f(x), z\| \geq \varepsilon) \\ &+ \delta(\{n \in \mathbb{N} : f_n(x) \neq g_n\}) \end{aligned} \tag{1}$$

Since $\lim_{n\to\infty} \|g_n(x), z\| = \|f(x), z\|$, for each $x \in X$ and each nonzero $z \in Y$. The set $\{n \in \mathbb{N} : \|g_n(x) - f(x), z\| \geq \varepsilon\}$ contain finite number of integers and so

$$\delta(\{n \in \mathbb{N} : \|g_n(x) - f(x), z\| \geq \varepsilon\}) = 0.$$

Using inequality (1) we get for every $\varepsilon > 0$

$$\delta(\{n \in \mathbb{N} : \|f_n(x) - f(x), z\| \geq \varepsilon\}) = 0,$$

for each $x \in X$ and each nonzero $z \in Y$ and so consequently

$$st - \lim_{n\to\infty} \|f_n(x), z\| = \|f(x), z\|.$$

Theorem 3.3. *Let $\alpha \in \mathbb{R}$. If for each $x \in X$ and each nonzero $z \in Y$,*

$$st - \lim_{n\to\infty} \|f_n(x), z\| = \|f(x), z\| \quad and \quad st - \lim_{n\to\infty} \|g_n(x), z\| = \|g(x), z\|,$$

then

(i) *$st - \lim_{n\to\infty} \|f_n(x) + g_n(x), z\| = \|f(x) + g(x), z\|$ and*
(ii) *$st - \lim_{n\to\infty} \|\alpha f_n(x), z\| = \|\alpha f(x), z\|$.*

Proof 3.3. *(i) Suppose that*

$$st-\lim_{n\to\infty}\|f_n(x),z\|=\|f(x),z\| \quad and \quad st-\lim_{n\to\infty}\|g_n(x),z\|=\|g(x),z\|$$

for each $x\in X$ and each nonzero $z\in Y$. Then, $\delta(K_1)=0$ and $\delta(K_2)=0$ where

$$K_1=K_1(\varepsilon,z):\left\{n\in\mathbb{N}:\|f_n(x)-f(x),z\|\geq\frac{\varepsilon}{2}\right\}$$

and

$$K_2=K_2(\varepsilon,z):\left\{n\in\mathbb{N}:\|g_n(x)-g(x),z\|\geq\frac{\varepsilon}{2}\right\}$$

for every $\varepsilon>0$, each $x\in X$ and each nonzero $z\in Y$. Let

$$K=K(\varepsilon,z)=\{n\in\mathbb{N}:\|(f_n(x)+g_n(x))-(f(x)+g(x)),z\|\geq\varepsilon\}.$$

To prove that $\delta(K)=0$, it suffices to show that $K\subset K_1\cup K_2$. Let $n_0\in K$ then, for each $x\in X$ and each nonzero $z\in Y$,

$$\|(f_{n_0}(x)+g_{n_0}(x))-(f(x)+g(x)),z\|\geq\varepsilon. \qquad (2)$$

Suppose to the contrary, that $n_0\notin K_1\cup K_2$. Then, $n_0\notin K_1$ and $n_0\notin K_2$. If $n_0\notin K_1$ and $n_0\notin K_2$ then, for each $x\in X$ and each nonzero $z\in Y$,

$$\|f_{n_0}(x)-f(x),z\|<\frac{\varepsilon}{2} \quad and \quad \|g_{n_0}(x)-g(x),z\|<\frac{\varepsilon}{2}.$$

Then, we get

$$\begin{aligned}\|(f_{n_0}(x)+g_{n_0}(x))-(f(x)+g(x)),z\| &\leq \|f_{n_0}(x)-f(x),z\|+\|g_{n_0}(x)-g(x),z\|\\ &< \frac{\varepsilon}{2}+\frac{\varepsilon}{2}\\ &= \varepsilon,\end{aligned}$$

for each $x\in X$ and each nonzero $z\in Y$, which contradicts (2). Hence $n_0\in K_1\cup K_2$ and so $K\subset K_1\cup K_2$.

(ii) *Let $\alpha\in\mathbb{R}$ ($\alpha\neq 0$) and for each $x\in X$ and each nonzero $z\in Y$,*

$$st-\lim_{n\to\infty}\|f_n(x),z\|=\|f(x),z\|.$$

Then, we get

$$\delta\left(\left\{n\in\mathbb{N}:\|f_n(x)-f(x),z\|\geq\frac{\varepsilon}{|\alpha|}\right\}\right)=0.$$

Therefore, for each $x\in X$ and each nonzero $z\in Y$, we have

$$\begin{aligned}\{n\in\mathbb{N}:\|\alpha f_n(x)-\alpha f(x),z\|\geq\varepsilon\} &= \{n\in\mathbb{N}:|\alpha|\|f_n(x)-f(x),z\|\geq\varepsilon\}\\ &= \left\{n\in\mathbb{N}:\|f_n(x)-f(x),z\|\geq\frac{\varepsilon}{|\alpha|}\right\}.\end{aligned}$$

Hence, the right hand side of above equality equals 0. Therefore, for each $x\in X$ and each nonzero $z\in Y$, we have

$$st-\lim_{n\to\infty}\|\alpha f_n(x),z\|=\|\alpha f(x),z\|.$$

Now, we give the concept of statistical Cauchy sequence and investigate relationships between statistical Cauchy sequence and statistical convergence in 2-normed space.

Definition 3.2. *The sequences of functions $\{f_n\}$ is said to be statistically Cauchy sequence, if for every $\varepsilon>0$ and each nonzero $z\in Y$, there exist a number $k=k(\varepsilon,z)$ such that*

$$\delta(\{n\in\mathbb{N}:\|f_n(x)-f_k(x),z\|\geq\varepsilon\})=0$$

for each $x\in X$ i.e.,

$$\|f_n(x)-f_k(x),z\|<\varepsilon, \quad a.a.\ n.$$

Theorem 3.4. *Let $\{f_n\}_{n\geq 1}$ be a statistically Cauchy sequence of functions in a finite dimensional 2-normed space $(X, \|.,.\|)$. Then, there exists a convergent sequence of functions $\{g_n\}_{n\geq 1}$ in $(X, \|.,.\|)$ such that $f_n = g_n$, for a.a. n.*

Proof 3.4. *First note that $\{f_n\}_{n\geq 1}$ is a statistically Cauchy sequence of functions in $(X, \|.\|_\infty)$. Choose a natural number $k(1)$ such that the closed ball $B_u^1 = B_u(f_{k(1)}(x), 1)$ contains $f_n(x)$ for a.a. n and for each $x \in X$. Then, choose a natural number $k(2)$ such that the closed ball $B_2 = B_u(f_{k(1)}(x), \frac{1}{2})$ contains $f_n(x)$ for a.a. n and for each $x \in X$. Note that $B_u^2 = B_u^1 \cap B_2$ also contains $f_n(x)$ for a.a. n and for each $x \in X$. Thus, by continuing of this process, we can obtain a sequence $\{B_u^m\}_{m\geq 1}$ of nested closed balls such that diam $(B_u^m) \leq \frac{1}{2^m}$. Therefore,*

$$\bigcap_{m=1}^{\infty} B_u^m = \{h(x)\},$$

where h is a function from X to Y. Since each B_u^m contains $f_n(x)$ for a.a. n and for each $x \in X$, we can choose a sequence of strictly increasing natural numbers $\{S_m\}_{m\geq 1}$ such that for each $x \in X$,

$$\frac{1}{n}|\{n \in \mathbb{N} : f_n(x) \notin B_u^m\}| < \frac{1}{m}, \quad if \quad n > S_m.$$

Put $R_m = \{n \in \mathbb{N} : n > S_m, \ \ f_n(x) \notin B_u^m\}$ for each $x \in X$, for all $m \geq 1$ and $R = \bigcup_{m=1}^{\infty} R_m$. Now, for each $x \in X$, define the sequence of functions $\{g_n\}_{n\geq 1}$ as following

$$g_n(x) = \begin{cases} h(x), & if \quad n \in R \\ f_n(x), & otherwise. \end{cases}$$

Note that, $\lim_{n\to\infty} g_n(x) = h(x)$, for each $x \in X$. In fact, for each $\varepsilon > 0$ and for each $x \in X$, choose a natural number m such that $\varepsilon > \frac{1}{m} > 0$. Then, for each $n > S_m$ and for each $x \in X$, $g_n(x) = h(x)$ or $g_n(x) = f_n(x) \in B_u^m$ and so in each case

$$\|g_n(x) - h(x)\|_\infty \leq diam(B_u^m) \leq \frac{1}{2^{m-1}}.$$

Since, for each $x \in X$, $\{n \in \mathbb{N} : g_n(x) \neq f_n(x)\} \subseteq \{n \in \mathbb{N} : f_n(x) \notin B_u^m\}$, we have

$$\frac{1}{n}|\{n \in \mathbb{N} : g_n(x) \neq f_n(x)\}| \leq \frac{1}{n}|\{n \in \mathbb{N} : f_n(x) \notin B_u^m\}| < \frac{1}{m},$$

and so

$$\delta(\{n \in \mathbb{N} : g_n(x) \neq f_n(x)\}) = 0.$$

Thus, $g_n(x) = f_n(x)$ for a.a. n and for each $x \in X$ in $(X, \|.\|_\infty)$. Suppose that $\{u_1, ..., u_d\}$ is a basis for $(X, \|.,.\|)$. Since, for each $x \in X$,

$$\lim_{n\to\infty} \|g_n(x) - h(x)\|_\infty = 0 \quad and \quad \|g_n(x) - h(x), u_i\| \leq \|g_n(x) - h(x)\|_\infty$$

for all $1 \leq i \leq d$, then we have

$$\lim_{n\to\infty} \|g_n(x) - h(x), z\|_\infty = 0,$$

for each $x \in X$ and each nonzero $z \in X$. It completes the proof.

Theorem 3.5. *The sequence $\{f_n\}$ is statistically convergent if and only if $\{f_n\}$ is a statistically Cauchy sequence of functions.*

Proof 3.5. *Assume that* f *be function from* X *to* Y *and* $st-\lim_{n\to\infty}\|f_n(x),z\| = \|f(x),z\|$ *for each* $x\in X$ *and each nonzero* $z\in Y$ *and* $\varepsilon>0$. *Then, for each* $x\in X$ *and each nonzero* $z\in Y$, *we have*

$$\|f_n(x)-f(x),z\|<\frac{\varepsilon}{2},\quad a.a.\ n.$$

If $k=k(\varepsilon,z)$ *is chosen so that for each* $x\in X$ *and each nonzero* $z\in Y$,

$$\|f_k(x)-f(x),z\|<\frac{\varepsilon}{2},$$

and so we have

$$\begin{aligned}\|f_n(x)-f_k(x),z\| &\le \|f_n(x)-f(x),z\|+\|f(x)-f_k(x),z\|\\ &< \frac{\varepsilon}{2}+\frac{\varepsilon}{2}\\ &= \varepsilon,\quad a.a.\ n.\end{aligned}$$

Hence, $\{f_n\}$ *is statistically Cauchy sequence of functions.*

Now, assume that $\{f_n\}$ *is statistically Cauchy sequence of function. By Theorem 3.4, there exists a convergent sequence* $\{g_n\}_{n\in\mathbb{N}}$ *from* X *to* Y *such that* $f_n=g_n$ *for a.a.* n. *By Theorem 3.2, we have*

$$st-\lim\|f_n(x),z\|=\|f(x),z\|$$

for each $x\in X$ *and each nonzero* $z\in Y$.

Now, as an immediate consequence of Theorem 3.2 we give the following theorem without the proof.

Theorem 3.6. *If* $st-\lim\|f_n(x),z\|=\|f(x),z\|$ *for each* $x\in X$ *and each nonzero* $z\in Y$, *then* $\{f_n\}_{n\in\mathbb{N}}$ *has a subsequence of function* $\{f_{n_i}\}$ *such that*

$$\lim_{i\to\infty}\|f_{n_i}(x),z\|=\|f(x),z\|$$

for each $x\in X$ *and each nonzero* $z\in Y$.

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