On Certain Operator Method for Solving Differential Equations

Batirkhan Turmetov

Akhmet Yasawi University, Turkistan, Kazakhstan
E-mail: turmetovbh@mail.ru

Abstract: In this paper a new method of construction of a solution for a wide class of fractional order differential equations is presented. This method is a generalization of the method of operator algorithms to fractional order differential equations. Let $m-1<\alpha\leq\gamma\leq m, m\in N$, $D^{\alpha,\gamma}f(t)=J^{(\gamma-\alpha)}\frac{d^m}{dt^m}J^{(m-\gamma)}f(t)$. Then the following proposition is true.

Theorem 1. Let $m-1<\alpha\leq\gamma\leq m, m=1,2,...,\ s=\gamma-1,...,\gamma-m$. Then the functions

$$y_{s,p}(t)=\sum_{i=p}^{\infty}\lambda^{i-p}\begin{pmatrix} i \\ p \end{pmatrix}\frac{t^{i\alpha+s}}{\Gamma(i\alpha+s+1)}$$

for all values of $p=0,1,...,n-1$ are solutions of the following equation of fractional order:

$$(D^{\alpha,\gamma}-\lambda)^n y(t)=0, t>0.$$

Let $E^p_{\alpha,\alpha}(\lambda,t)=\sum\limits_{i=p}^{\infty}\lambda^{i-p}\begin{pmatrix} i \\ p \end{pmatrix}\frac{t^{i\alpha}}{\Gamma(i\alpha+\alpha)}, p=0,1,....$ Consider the following function

$$y_p(f)(t)=\int\limits_0^t(t-\tau)^{\alpha-1}E^p_{\alpha.\alpha}(-\lambda(t-\tau)^\alpha)f(\tau)d\tau,$$

Theorem 2. Let $m-1<\alpha\leq\gamma\leq m, m=1,2,....$ Then functions $y_p(f)(t), p=0,1,...$ form f-normalized system with respect to the operator $D^{\alpha,\gamma}-\lambda$, i.e. the following equality holds:

$$\begin{cases} (D^{\alpha,\gamma}-\lambda)\,y_0(f)(t)=f(t), \\ (D^{\alpha,\gamma}-\lambda)\,y_p(f)(t)=f_{p-1}(t),\ \ p\geq 1 \end{cases}$$

Keywords: operator method, operator of fractional differentiation and integration, fractional order differential equation

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Asymptotically $\mathcal{I}$-Cesàro Equivalence of Sequences of Sets

Uğur Ulusu

Department of Mathematics, Afyon Kocatepe University, Afyonkarahisar, Turkey
E-mail: ulusu@aku.edu.tr

Abstract: In this study, we defined concept of asymptotically $\mathcal{I}$-Cesàro equivalence and investigate the relationship between the concepts of asymptotically strongly $\mathcal{I}$-Cesàro equivalence, asymptotically strongly $\mathcal{I}$-lacunary equivalence, asymptotically p-strongly $\mathcal{I}$-Cesàro equivalence and asymptotically $\mathcal{I}$-statistical equivalence of sequences of sets.

Keywords: Asymptotically equivalence, Cesàro summability, lacunary sequence, Ideal convergence, Sequences of sets, Wijsman convergence.

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